Math 220 – Business Calculus
Spring 2013
Test 5

Name Answer Key

Version A

1. For these questions, the interest rate is 5% per year paid continuously. For a) and b) you may leave your answer un-simplified. For c), give an answer in dollars.

a) How much will an investment made continuously at a rate of \$900 per year be worth in 7 years?

$$e^{(.05)7}\int_0^7 900\, e^{-.05t}\,dt = e^{.35}\, 900 \frac{e^{-.05t}}{-.05}\Big|_0^7$$

$$= \frac{900}{-.05} e^{.35}\left(e^{-.35} - e^0\right)$$

b) What is the present value of an annuity that pays \$4000 per year for 12 years?

$$\int_0^{12} 4000\, e^{-.05t}\,dt = \frac{4000}{-.05} e^{-.05t}\Big|_0^{12}$$

$$= \frac{4000}{-.05}\left(e^{-.60} - e^0\right)$$

c) What is the capital value of a piece of property that is expected to provide income of $\$6300e^{0.02t}$ per year forever? (Where t is the number of years from today)

$$\int_0^{\infty} 6300\, e^{.02t} e^{-.05t}\,dt = \lim_{V\to\infty}\int_0^V 6300\, e^{-.03t}\,dt$$

$$= \lim_{V\to\infty} \frac{6300}{-.03}\left(e^{-.03}\right)\Big|_0^V = \lim_{V\to\infty} 210000\left(e^{-.03V} - e^0\right)$$

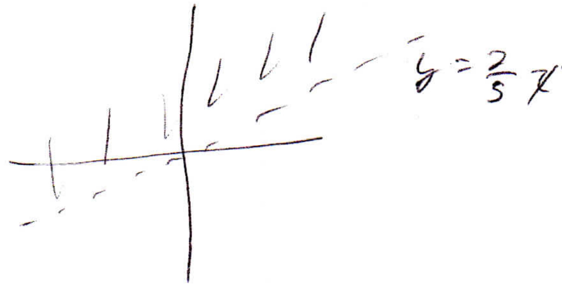
$$= \$210{,}000$$

2. Find the domain of each function and sketch it.

a) $\ln(5y - 2x)$

$5y - 2x > 0$

$y > \frac{2}{5}x$



b) $\frac{x+1}{3x+5y}$

$3x + 5y \neq 0$

$y \neq -\frac{3}{5}x$

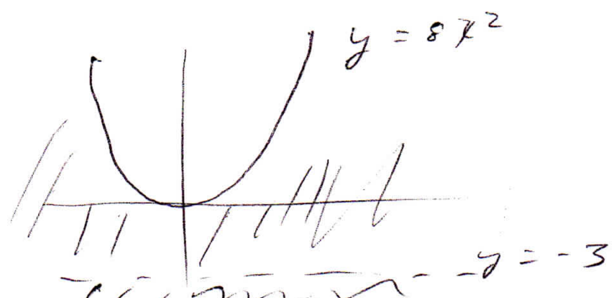
c) $\frac{\sqrt{8x^2-y}}{y+3}$

$y \neq -3$

and

$8x^2 - y \geq 0$

$y \leq 8x^2$



3. Find the indicated partial derivatives of each function

a) $f(x,y) = 4x^3 - 5x^4y^2 + 7y^3$ $\qquad f_x(x,y) = 12x^2 - 20x^3y^2$

b) $z = \ln(6x^2 - 4y^3)$ $\qquad \frac{\delta^2 z}{\delta y^2} = \frac{(6x^2-4y^3)(-24y) - (-12y^2)(-12y^2)}{(6x^2-4y^3)^2}$

$\frac{dz}{dy} = \frac{-12y^2}{6x^2-4y^3}$

c) $w = e^{36y+x^2}$ $\qquad \frac{\delta w}{\delta y} = (e^{36y+x^2})(36)$

d) $g(s,t) = \sqrt[3]{3s^3 + 7t^4}$ $\qquad g_s(s,t) = \frac{1}{3}(3s^3+7t^4)^{-2/3}(9s^2)$

4. The profit from selling tables and chairs is given by the function P(t,c) = 40t + 16c + 45/c. Use derivatives to estimate the increased profit when 12 tables are being sold and the number of chairs being sold increases from 30 to 31. Give your answer to the nearest penny. No other method will be accepted. However, you may use another method to check to see if your answer is close to the exact answer.

$P_c = 16 - \frac{45}{c^2}$

$P_c(30) = 16 - \frac{45}{900} = \$15.95$

$\frac{5}{100}$

5. Given this function: $f(x,y) = 4(3-x)^2 + \frac{7}{3}y^3 + \frac{7}{2}y^2 - 42y$, find any minima, maxima or saddle points that exist.

$f_x = 8(3-x)(-1) = 0$ $\quad x = 3$

$f_y = 7y^2 + 7y - 42$

$7(y^2 + y - 6) = 0$

$7(y+3)(y-2) = 0$ $\quad y = -3 \text{ or } 2$

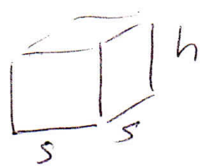
$f_{xx} = 8$

$f_{yy} = 14y + 7$

$f_{xy} = 0$

x	y	f_{xx}	f_{yy}	f_{xy}	D	
3	-3	8	-35	0	-280	saddle
3	2	8	35	0	+280	min

6a. Find the maximum volume of a box with a square base that can be built subject to this constraint. The cost of the box is \$5 times the length of a side (s) plus \$4 times the height (h). The money available to build the box is \$54. Use Lagrange multipliers. No other solution will be accepted.



$V = s^2 h$

$C = 5s + 4h$

$F(s, h, \lambda) = s^2 h + \lambda(5s + 4h - 54)$

$F_s = 2sh + 5\lambda = 0 \Rightarrow 8sh + 20\lambda = 0$

$F_h = s^2 + 4\lambda = 0 \Rightarrow 5s^2 + 20\lambda = 0$

$8sh - 5s^2 = 0$

$s(8h - 5s) = 0$

$s \neq 0$ or $8h = 5s$

$4h = \frac{5}{2}s$

$F_\lambda = 5s + 4h - 54 = 0$

$5s + \frac{5}{2}s = 54$

$\frac{15}{2}s = 54$

$s = \frac{108}{15} = \frac{36}{5}$

$h = \frac{5(108)}{8(15)} = \frac{27}{6} = \frac{9}{2}$

$V = \frac{(36)^2}{5^2} \cdot \frac{9}{2} = 233.28$

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Name Answer Key

Version B

1. For these questions, the interest rate is 4% per year paid continuously. For a) and b) you may leave your answer un-simplified. For c), give an answer in dollars.

a) How much will an investment made continuously at a rate of \$600 per year be worth in 8 years?

$$e^{.04(8)} \int_0^8 600e^{-.04t}\,dt = e^{.32}\, 600 \frac{e^{-.04t}}{-.04}\Big|_0^8$$

$$= \frac{600e^{.32}}{-.04}\left(e^{-.32} - e^{0}\right)$$

b) What is the present value of an annuity that pays \$7000 per year for 18 years?

$$\int_0^{18} 7000e^{-.04t}\,dt = 7000\frac{e^{-.04t}}{-.04}\Big|_0^{18}$$

$$= \frac{7000}{-.04}\left(e^{-.72} - e^{0}\right)$$

c) What is the capital value of a piece of property that is expected to provide income of \$7200$e^{0.02t}$ per year forever? (Where t is the number of years from today)

$$\int_0^{\infty} 7200e^{.02t}e^{-.04t}\,dt = \lim_{V\to\infty}\int_0^{V} 7200e^{-.02t}\,dt$$

$$= \lim_{V\to\infty} \frac{7200}{-.02}\left(e^{-.02t}\right)\Big|_0^{18}$$

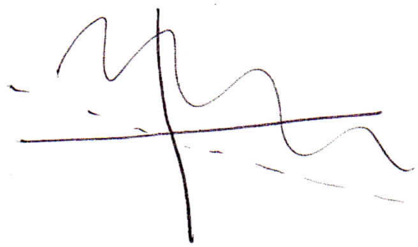
$$= \lim_{V\to\infty} 360000\left(e^{-.02V} - e^{0}\right) = \$360,000$$

2. Find the domain of each function and sketch it.

a) $ln(4y + 3x)$

$4y + 3x > 0$

$y > -\frac{3}{4}x$

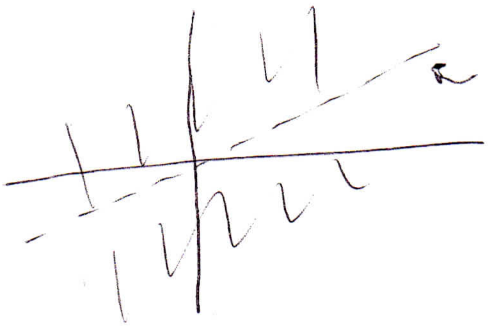


$-y = -3/4\, y$

b) $\frac{x+1}{2x-4y}$

$2x - 4y \neq 0$

$y \neq \frac{1}{2}x$



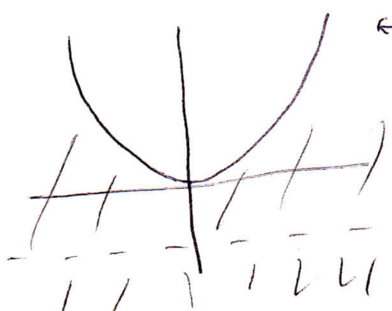
$y = 1/2\, x$

c) $\frac{\sqrt{10x^2-y}}{y+2}$

$y \neq -2$

and $10x^2 - y \geq 0$

$y \leq 10x^2$



$\leftarrow y = 10x^2$

$y = -2$

3. Find the indicated partial derivatives of each function

a) $f(x,y) = 4x^3 - 5x^4y^2 + 7y^3$ $\qquad f_y(x,y) = -10x^4y + 21y^2$

b) $z = \ln(5x^2 - 2y^3)$ $\qquad \frac{\delta^2 z}{\delta x^2} = \frac{(5x^2-2y^3)(10) - (10x)(10x)}{(5x^2-2y^3)^2}$

$\frac{dz}{dx} = \frac{10x}{5x^2-2y^3}$

c) $w = e^{25x^2+y}$ $\qquad \frac{\delta w}{\delta x} = (e^{25x^2+y})(50x)$

d) $g(s,t) = \sqrt[4]{3s^3 + 7t^4}$ $\qquad g_t(s,t) = \frac{1}{4}(3s^3 + 7t^4)^{-3/4}(28t^3)$

4. The profit from selling tables and chairs is given by the function P(t,c) = 50t + 17c + 16/c. Use derivatives to estimate the increased profit when 12 tables are being sold and the number of chairs being sold increases from 20 to 21. Give your answer to the nearest penny. No other method will be accepted. However, you may use another method to check to see if your answer is close to the exact answer.

$P_c = 17 - \frac{16}{c^2}$

$P_c(20) = 17 - \frac{16}{400} \doteq \$16.96$

$\uparrow$

$\frac{4}{100}$

5. Given this function: $f(x, y) = 3(4 - x)^2 + \frac{5}{3}y^3 + \frac{10}{2}y^2 - 40y$, find any minima, maxima or saddle points that exist.

$f_x = 6(4 - x)(-1) = 0 \qquad x = 4$

$f_y = 5y^2 + 10y - 40 = 0$

$5(y^2 + 2y - 8) = 0$

$5(y + 4)(y - 2) = 0 \qquad y = -4 \text{ or } y = 2$

$f_{xx} = 6$

$F_{yy} = 10y + 10$

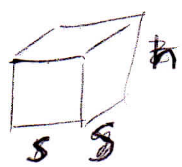
$F_{xy} = 0$

$\cancel{D(4, -4) =}$

x	y	fxx	fyy	fxy	D	
4	-4	6	-30	0	-180	saddle
4	2	6	30	0	180	min

6a. Find the maximum volume of a box with a square base that can be built subject to this constraint. The cost of the box is \$2 times the length of a side (s) plus \$3 times the height (h). The money available to build the box is \$56. Use Lagrange multipliers. No other solution will be accepted.

$V = \cancel{x^2}\ s^2 h$



$C = 2s + 3h = 56$

$F(s, h, \lambda) = s^2 h + \lambda(2s + 3h - 56)$

$F_s = 2sh + 2\lambda = 0 \qquad 6sh + 6\lambda = 0 \qquad 6sh = 2s^2$

$F_h = s^2 + 3\lambda = 0 \qquad 2s^2 + 6\lambda = 0 \qquad 2s(3h - s) = 0$

$\cancel{s = 0}$ or

$Vol = 0 \qquad 3h = s$

$F_\lambda = 2s + 3h - 56 = 0$

$2s + s - 56 = 0$

$s = \frac{56}{3} \qquad h = \frac{56}{9}$