Math 330 Section 3 Homework 02

Written assignments:
First submission: Wednesday, September 9
Last submission: Thursday, September 24

Correction to both written assignments:
Sept 9, 2015, 8:40 PM: In both cases, I wrote "... up to but NOT including prop.2.4 ...". I meant to say "... up to but AND including prop.2.4 ...". Fair warning: I do not believe that prop.2.4 will be of any help.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook: all of ch.1

Other course material:
Nothing so far

New reading assignments:

Reading assignment 1 - due: Tuesday, September 8 Read carefully ch.2.1 and ch.2.2. in the book.

Be prepared to take a quiz which asks for everything asked for in previous assignments PLUS
axiom 2.1, the definitions of $\mathbb{N}$, of **natural numbers**, **positive integers**, **negative integers**,
the definitions of **Law of the Excluded Middle**, $a < b$, $a \leq b$, $a > b$, $a \geq b$,
less than, **less than or equal to**, **greater than**, **greater than or equal to**,
the definitions of **equality of sets**, $A \subseteq B$, $A \supseteq B$.

Core propositions you should be able to cite:
Prop.2.2 (Trichotomy), Prop.2.3 ($1 \in \mathbb{N}$), Prop.2.13 ($\mathbb{N} = \{n \in \mathbb{Z} : n > 0\}$)

Reading assignment 2 - due: Wednesday, September 9 Read carefully ch.2.3 on induction in the book. Read everything starting with thm.2.17 up to and including the "template for proofs by induction" **extra carefully and repeatedly**! I shall nickle-and-dime you on correctly writing down proofs that use induction repeatedly and it is a **100% certainty** that such proofs will appear on on some of the exams and the final!

Be prepared to take a quiz which asks for everything asked for previously PLUS
axiom 2.15, all theorems (thm.2.17 and 2.25) and their names,
all definitions: **digits**, **base case**, **inductions step**
and all core propositions: 2.14, 2.16, 2.20 - 2.22

Some advice: Don't skip the margins on to of p.20 (the "ladder principle") because it is a good way of deepening your intuitive grasp of how proofs by induction work.

Reading assignment 3 - due: Friday, September 11 Read carefully all of ch.2 in the book (the additional chapter 2.4 has less than 2 pages).

Be prepared to take a quiz which asks for everything asked for previously PLUS
the one theorem (thm.2.32) and its name and props 2.33, 2.34.
The book only contains one definition: that of a **smallest element** or **minimum** of a set.
Here are three more which you also must be prepared to answer:

Definition Let $A \subset \mathbb{Z}$ and let $m_0 \in \mathbb{Z}$.

a. If m_0 satisfies $m_0 \leq a$ for all $a \in A$ then we call m_0 a **lower bound** of A
b. If m_0 satisfies $m_0 \geq a$ for all $a \in A$ then we call m_0 an **upper bound** of A
c. If both m_0 is an **upper bound** of A and also $m_0 \in A$ then we call m_0 the maximum of A and we write $m_0 = \max(A)$.

You should understand that min(A), as defined in the text, can be described as follows:

d. $min(A)$, if it exists, is a **lower bound** of A which also is an element of A.

Written assignment 1:

Use everything up to, AND including prop.2.4, to prove that if $k \in \mathbb{Z}$ then $k + 1 > k$. Hint: Use prop.2.3.

Written assignment 2:

Use assignment 1 above and everything up to AND including prop.2.4, to prove prop.2.5: For each $n \in \mathbb{N}$ there exists $M \in \mathbb{N}$ such that $M > n$.

GOOD NEWS: When you do assignments from chapter 2 and later chapters you need no longer justify the rules of arithmetic given to you in ch.1. You may even use the "general laws of associativity": Given any finite sum of integers such as $(m_1 + m_2) + (n_1 + n_2)$ you may regroup the parentheses and even drop them. The same is true for products.