Math 330 Section 3 Homework 05

Written assignments:
First submission: Monday, September 28
Last submission: Monday, October 12

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook: all of ch.1 - 3 and ch.4.1 - 4.3

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2"

New reading assignments:

Reading assignment 1 - due: Friday, September 25

a. Read carefully the remainder of ch. 4 in the book. I intend to finish this chapter on Friday.

b. Read carefully ch.5.1 - 5.3 in the book. You have seen most of the material, if not all of it, in the two papers Sets 1 and Sets 2. This time the emphasis is on digging into the proofs as you will be asked in your next homework to do some formal proofs on sets.

What's important for quizzes and exams:
1. As usual, you must know and be able to recite all definitions so far encountered in B/G.
2. the connection between induction and recursion (thm.4.4, p.35) and the differences/similarities between "ordinary" and "strong" or "second form" induction.
3. The properties of finite series (prop.4.15-prop.4.18).
4. All theorems, propositions and corollaries in ch.4.4 (binomial theorem).

Reading assignment 2 - due: Friday, September 25

a. Click the link "Lecture Notes: Math 330 - Additional Material" in the course materials page of the web site and read chapter 4.1. It is less than one page (it only contains the "inclusion lemma" for sets) and does not depend on anything else. **You will read more and more items in this paper as time progresses.** In the final weeks of this course it will become your primary resource.

b. Click the link "Functions part 1" and read it according to the guidelines in the Additional course materials web page. You can count on having to turn in a "graded only once" homework centered on the exercises in that paper.

Written assignment 1:

Prove Prop. 4.6(iii) using induction: Given the definition of "Power" between props 4.5 and 4.6, prove that

$$(b^m)^k = b^{mk}$$

for $b \in \mathbb{Z}$ and $m, k \in \mathbb{Z}_{\geq 0}$.

You may use everything up to and including Prop.4.6(ii) but you won't need anything beyond the induction principle in ch.2., except for the proof of Prop.4.6(ii). That one provides an excellent template after which you can model your own proofs using induction.

Written assignment 2:

Prove Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including Prop.4.7.

Written assignment 3:

Prove Prop. 4.16(i) using induction: Let $(x_j)_{j\in\mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \leq b < c$. Then

$$\sum_{j=a}^{c} x_j = \sum_{j=a}^{b} x_j + \sum_{j=b+1}^{c} x_j.$$

For this proof we modify the definition of Σ (B/G p.34, 35) as I discussed in class: Let $a, n \in \mathbb{Z}$ and $a \leq n$.

$$\text{(i)} \quad \sum_{j=a}^{a} x_j = x_a, \qquad \text{(ii)} \quad \sum_{j=a}^{n+1} x_j = \sum_{j=a}^{n} x_j + x_{n+1}.$$

Hints: a) Do induction on c. b) Think carefully about the base case.