Math 330 Section 3 Homework 07

Corrected on Oct 8, 2015, 10:25 PM:

Originally assignment 2 displayed the following formula: $[x]_f \ = \ \{f(\tilde{x}) : \tilde{x} \in X \text{ and } f(\tilde{x}) = f(x)\}$.
This was corrected as follows: $[x]_f \ = \ \{\tilde{x} : \tilde{x} \in X \text{ and } f(\tilde{x}) = f(x)\}$.

Written assignments:
First submission: Monday, October 5
Last submission: Monday, October 19

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook: all of ch.1 - 5

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2", "Lecture Notes: Math 330 - Additional Material" ch.4.1 (Inclusion Lemma)

New reading assignments:

Reading assignment 1 - due: Friday, October 2

Read carefully ch. 6 in the book about the basics of equivalence relations (3 pages). There is only one proof (prop.6.4) and you should make an effort to understand it completely and be prepared to ask questions in class about the spots where you got lost. I shall lecture about this on Friday, so it is to your advantage if you are well prepared.

Written assignments:

Written assignment 1:

Let $X, Y \neq \emptyset$ and $f : X \longrightarrow Y\}$ be a function. Define a relation $R \subseteq X \times X$ as follows:

$$R \ := \{(x_1, x_2) : x_1, x_2 \in X \text{ and } f(x_1) = f(x_2)\}.$$

Prove that R defines an equivalence relation on X.

Definition: The equivalence classes $[x]$ of the equivalence relation defined above are denoted by $[x]_f$. $[x]_f$ is called the **fiber over** $f(x)$.

Written assignment 2: Prove the following for the equivalence relation defined in assignment 1:

If $x \in X$ then $[x]_f \ = \ \{\tilde{x} : \tilde{x} \in X \text{ and } f(\tilde{x}) = f(x)\}$.

NOTE: there was a mistake in the above formula in the original homework posting.

Hint: remember that you prove equality of two sets by showing that each one is a subset of the other.

Written assignment 3:

Let $X, Y \neq \emptyset$ and $f : X \longrightarrow Y\}$ be a function. Again, we look at the equivalence relation defined in assignment 1. Let $\emptyset \neq A \subseteq X$. Prove that $f^{-1}\big(f(A)\big) \ = \ \bigcup\limits_{a \in A} [a]_f$

Note: Here you have an example for the usefulness of a "general index set" (in the formula above: A). to properly formulate a mathematical statement.

Hint: To get a feeling for what is involved, look what the left side and the right side give you for the following examples:
a. $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined as $f(x) = x^2$; choose $A = [-1, 2]$ (the closed interval).
b. $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined as $f(x) = x^2$; choose $A = \{0, \pm\pi/2, \pi\}$
c. A simple case: What are the fibers of an injective function?