Math 330 Section 3 Homework 08

Correction on Oct 14: Error in hints for 1a: uniqueness: I amended $m = \tilde{q} + \tilde{r}$ to $m = \tilde{q}n + \tilde{r}$.

Update on Oct 10: Hints for doing 1b by induction on m rather than using the Well-Ordering principle.

Update on Oct 8: Written assignment 1 has been split into 1a and 1b and hints were added.

Written assignments:
First submission: Friday, October 9
Last submission: Friday, October 23

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook: all of ch.1 - 5 and ch.6.1

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2",
"Lecture Notes: Math 330 - Additional Material":
ch.4.1 (Inclusion Lemma)

New reading assignments:

Reading assignment 1 - due: Monday, October 5

a. Read carefully ch.4 in my "Math 330 - Additional Material" notes. This will be of great help for at least one of the exam problems and the homework #7 assignment.

b. Read carefully ch.6.2 of B/G: "The Division Algorithm". No proofs are given and it's less than 1 1/2 pages. Like for most of ch.4, it is important that understand what the theorems and propositions mean. Two pitfalls:

b1. The zero polynomial does NOT have degree zero. Rather, a degree is not defined for it. The polynomials of degree zero are the constant polynomials $p(x) = c$ for which $c \neq 0$!

b2: Proposition 6.18 is in error. The division algorithm needs divisibility defined for the coefficients of a polynomial and you don't have that for coefficients in $\mathbb{Z}$. The polynomials must be allowed to have rational or real (or complex) coefficients!

Reading assignment 2 - due: Tuesday, October 6

a. Read ch.3. in my "Math 330 - Additional Material" notes. The emphasis is not on proofs but, just as the additional course material on sets and functions, I want you to understand the material presented there. Nothing in this chapter should be new to you and it will help solidify your knowledge of sets and functions and also with ch.4 of that document (due the previous day).

b. Read carefully ch.6.3 and ch.6.4 in the B/G book. It is a certainty that material from ch.6 will be on exam 2 and the final (and on some quizzes).

c. Read carefully ch.9.1(!) in the B/G book. It contains additional material on injectivity, surjectivity and bijectivity. I shall spend some of Tuesday's lecture on the material in ch.9.1 that you have not seen in the additiional course material. Understanding this material better will come in handy for Wednesday's exam 1, but note that what you need to know about ch.9.1 you were taught in lecture and the readings from the additional course material.

Written assignments:

Written assignment 1: Prove Thm.6.13, p.59 of B/G (Division algorithm): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

Do this in two separate steps: you'll get separate credit for each one.

Problem 1a: Uniqueness of q, r:

Prove uniqueness of the "decomposition" $m = qn + r$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Problem 1b: Existence of q, r:

Much harder: Prove the existence of q and r.

Hints: Review the Well-Ordering principle from ch.2. It will give the easiest way to prove this theorem. Remember we extended the well-ordering principle as follows: Let $A \subseteq \mathbb{Z}$ have lower bounds (which is especially true if $A \subseteq \mathbb{N}$ or $A \subseteq \mathbb{Z}_{\geq 0}$). Then A has a minimum.

Apply this to the set

$$A \;:=\; A(m,n) \;:=\; \{x \in \mathbb{Z}_{\geq 0} : x = m - kn \text{ for some } k \in \mathbb{Z}\}$$

Why: $\min(A)$ will give you the remainder r (do some numbers for yourself to see why that should be so). Note that you must prove that A is not empty, something you might do differently for $m \geqq 0$ and $m < 0$.

ADDED ON OCT 10: Alternate proof of existence: If you like to do a "straight" proof by induction rather than work with the Well-Ordering principle:

a. handle $m \geq 0$ first and do induction on m (NOT on n).

b. When that is done then assume $m < 0$ and apply the fact (you've just proved) that $-m$ nas a decomposition $-m = nq + r$ for some suitable q, r: you'll get $m = (-q)n + (-r)$. You are not there yet because $-r$ does not satisfy $0 \leqq (-r) < n$ and you must adjust. Advice: use some scratch paper and see what you get for $m = -23$ and $n = 5$.

Written assignment 2: Prove Thm.6.16, p.59 of B/G: Let $n \in \mathbb{Z}$. Then either n is even or $n + 1$ is even.

Hint: Apply the division algorithm with $n = 2$.