# Math 330 Section 3 Homework 09

*Due date: Tuesday, October 13, 2015* *Running total: 33 points*
*Last submission: Tuesday, October 27, 2015*

**Status - Reading Assignments:**

Here is the status of the reading assignments you were asked to complete by this date.

Textbook:
all of ch.1 - 6, ch.9.1

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2"

"Lecture Notes: Math 330 - Additional Material":
All of ch.3 (understand the material) all of ch.4 (understand the proofs!)

**New reading assignments:**

a. Read carefully ch.7.1 of B/G.

b. Optional: Read ch.7.2 (the addition algorithm) to the point that you understand "the algorithm" described on p.70 and the meaning of thm.7.17. I shall not lecture about ch.7.2 and you will not be at a disadvantage if you skip this section. If you are studying computer science it would be in your interest to dig into this material as algorithms and the ability to examine them from a "mathy" point of view are valuable for you.

**Assignment 1:**

Prove Prop.7.1 using induction: If $n \in \mathbb{N}$ then $n < 10^n$. You may use the fact that $10$ (defined as $9+1$) satisfies $0 < 1 < 2 < 10$. Justify your inequalities referring to prop. 2.7(i) - 2.7(iv).

**Assignment 2:**

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A := A(n) := \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) := \min(A).$$

Prop.7.3 states that, for all $n \in \mathbb{N},\ \nu(n) = k \iff 10^{k-1} \leqq n < 10^k$.

**Assignment 2a:** Prove "$\Rightarrow$" of prop.7.3.

**Assignment 2b:** Prove "$\Leftarrow$" of prop.7.3.

The math for assignment 2 is easy but you may find it hard to write down a proof that meets my demands for precision.

Hints: 1) I gave the set a name ($A$) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** $A$: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)