Math 330 Section 3 Homework 13

Update: Assignment 3, We assume $\alpha \in \mathbb{R}_{\geq 0}$ (not $\alpha \in \mathbb{R}_{>0}$) (Nov 12, 2015)
Update: Assignment 2, Hints: $\varepsilon := d(L, L')/2$ (not $d(L, L')$) (Nov 10, 2015)
Update: Assignment 3 added on Nov 9, 2015

Due date: Monday, November 2, 2015 *Running total: 42 points*
Last submission Monday, November 16, 2015

Textbook:
all of ch.1 - 8 (ch.7.2 was optional), ch.9, ch.10.1-10.4

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2"

"Lecture Notes: Math 330 - Additional Material":
All of ch.3 (understand the material)
all of ch.4 (understand the proofs!)
all of ch.5.1,
ch.5.2 up to and including definition 5.13 (liminf and limsup of a sequence). All of ch.6 (understand the material) EXCEPT ch. 6.2.2 on normed vector spaces
Subchapter 7.1.1: "Measuring the distance of real functions"

New reading assignment 1 - Due Mon, Nov.2:

"Lecture Notes: Math 330 - Additional Material":
Finish chapter 5 according to the following instructions: No need to try to understand the proofs of prop 5.3, cor.5.2, prop.5.4, prop.5.5, theorem 5.1. BUT understand the proof of thm.5.2 (Characterization of limits via limsup and liminf).
All definitions in ch.5 (and there is quite a lot of them) and theorems 5.1 and 5.2 are fair game for quizzes and exams. Here is my advice: Let $x_n := (-1)^n \cdot n/(n+1)$. Check the plausibility of the propositions and theorems of ch.5.2 by replacing general x_n with this specific sequence. Draw pictures of the "graph" of x_n like I did in class.

New reading assignment 2 - Due Tue, Nov.3:

B/G ch.10.5 (i.e., finish ch.10).

New reading assignment 3 - Due Wed, Nov.4:

B/G ch.11

Assignment 1:

Prove Prop.10.13(ii): $\lim\limits_{k\to\infty} \dfrac{k-1}{k} = 1$.

Assignment 2: Prove Prop.10.14: for sequences in abstract metric spaces: Let (X, d) be a metric space and let $L, L' \in X$ and assume that both $\lim\limits_{k\to\infty} x_k = L$ and $\lim\limits_{k\to\infty} x_k = L'$. Then $L = L'$.

Hints: A. Assume that $L \neq L'$ and let $\varepsilon := d(L, L')/2$. Use the triangle inequality to show that the two "ε-neighborhoods" $B_\varepsilon(L) := \{x \in X : d(x, L) < \varepsilon\}$ and $B_\varepsilon(L') := \{x \in X : d(x, L') < \varepsilon\}$ are disjoint and use the definition of a limit to show that all but at most finitely many of the x_n must belong simultaneously to both those neighborhoods.

B. The result of assignment #3 below might be helpful.

Assignment 3: Prove the following simplified version of B/G prop.10.11, using only material from B/G up to the end of ch.10.2 (Absolute Value): Let $\alpha \in \mathbb{R}_{\geq 0}$. Then $\alpha = 0 \iff \alpha < \epsilon$ for all $\varepsilon \in \mathbb{R}_{>0}$.

Hint: If $\alpha > 0$, what strictly positive number is less than α?