Math 330 Section 3 Homework 16

Update on 2015-11-30: The last submission date has been changed from Mon, 11/30 to Tue, 12/1 as I had forgotten to bring the graded homework to class.

Update on 2015-11-23: The running total has been decreased from 50 to 49 because #3 is an ALTERNATIVE to #1,2 and it will count 2 points if you solve it, BUT: you'll either get credit for #1,2 or for #3!

Due date: Mon, November 16, 2015 *Running total:* 49 *points*
Last submission Tue, December 1, 2015

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook:
all of ch.1 - ch.6, ch.7.1,
all of ch.8 - ch.13

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2"

"Lecture Notes: Math 330 - Additional Material":
All of ch.3 (understand the material)
all of ch.4 (understand the proofs!)
all of ch.5 (learn all definitions and the two theorems at the end, skip the proofs
All of ch.6 (understand the material) EXCEPT ch. 6.2.2 on normed vector spaces
Chapter 7 until end of subchapter 7.1.1: "Measuring the distance of real functions"

New reading assignments:

Reading assignment 1: due Mon, 11/16/2015

"Lecture Notes: Math 330 - Additional Material":
ch.3.4 (countable sets - understand the material well!)
ch.7.1.2-7.1.4 (understand the proofs but skip ch.7.1.4 starting at def. 7.12)

Reading assignment 2: due Wed, 11/18/2015

"Lecture Notes: Math 330 - Additional Material":
ch.7.1.5 (understand the proofs)

Reading assignment 3: due Fri, 11/20/2015

"Lecture Notes: Math 330 - Additional Material":
Remainder of ch.7: ch.7.1.6-ch.7.1.7 (understand the proofs)

Written assignments: Be sure to read the notes following assignment 3 before you start!

Assignment 1:

Let $\vec{v} = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ be an n–dimension vector. The ***Euclidean norm*** $\|\vec{v}\|$ of $\vec{v}$ was defined in Definition 6.3 "(Euclidean norm)" as follows:

$$\|\vec{v}\| \quad = \quad \sqrt{{x_1}^2 + {x_2}^2 + \ldots + {x_n}^2} \quad = \quad \sqrt{\sum_{j=1}^{n} {x_j}^2}.$$

Prove Example 7.3 ($\mathbb{R}^N : d(\vec{x}, \vec{y}) =$ Euclidean norm):

$$d(\vec{x}, \vec{y}) \; = \|\vec{x} - \vec{y}\| \; = \; \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + \ldots + (y_N - x_N)^2} \; = \; \sqrt{\sum_{j=1}^{N} (y_j - x_j)^2}$$

defines a metric on $\mathbb{R}^N$.

You are ALLOWED and ADVISED to use Proposition 6.1 (Properties of the Euclidian norm).

Assignment 2:

Prove prop.7.2 (Metric properties of the distance between real functions):
Let X be an arbitrary, non–empty set. Let

$$\mathscr{B}(X, \mathbb{R}) := \{h(\cdot) : h(\cdot) \text{ is a bounded real function on } X\}.$$

Let $f(\cdot), g(\cdot), h(\cdot) \in \mathscr{B}(X, \mathbb{R})$ Then the distance function

$$d(\cdot) : \; \mathscr{B}(X, \mathbb{R}) \times \mathscr{B}(X, \mathbb{R}) \longrightarrow \mathbb{R} \qquad (h_1, h_2) \longmapsto d(h_1, h_2) := \|h_1 - h_2\|$$

is a metric.

You are ALLOWED and ADVISED to use prop.7.1 (Properties of the norm of a real function) which you will find just before prop.7.2.

ALTERNATE Assignment 3 - counts TWO points:

Assignments 1 and 2 can be solved separately or you can solve them in a single attempt as follows:

Prove Theorem 7.1 (Norms define metric spaces).

Then refer to the propositions (find them!) in the document that state that the metrics in both assignments are derived from norms. Copy those propositions into your homework rather than simply referencing the number as those references are subject to change!

Note 1: In both assignments 1 and 2 it is trivial to prove positive definiteness and symmetry. To prove the triangle inequality, you want to use the triangle inequality for norms plus the fact that the norm is never negative. If you try to do it directly, you might be in for a lot of grief. If you do assignment 3 instead, the problem does not pose itself as you have no choice other than using the triangle inequality for norms.

Note 2: Assignments 1 and 2 can be solved separately or you can solve them in a single attempt as follows: Prove Theorem 7.1 (Norms define metric spaces). Then refer to the propositions in the document that state that the metrics in both assignments are derived from norms.