Math 330 Section 3 Homework 17

Due date: Tue, December 1, 2015 *Running total:* 53 *points*
Last submission Tue, December 15, 2015

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

Textbook:
all of ch.1 - ch.6, ch.7.1,
all of ch.8 - ch.13

Other course material (course materials page):
"Logic part 1", "Sets part 1", "Sets part 2", "Functions part 1", "Functions part 2"

"Lecture Notes: Math 330 - Additional Material":
All of ch.3 (understand the material)
all of ch.4 (understand the proofs!)
all of ch.5 (learn all definitions and the two theorems at the end, skip the proofs
All of ch.6 (understand the material) EXCEPT ch. 6.2.2 on normed vector spaces
all of ch.7.1, but skip the end of ch.7.1.4, starting at def. 7.12

New reading assignments:

Reading assignment 1: due Mon, 11/30/2015

"Lecture Notes: Math 330 - Additional Material":
Ch.7.2: Continuity

Reading assignment 2: due Tue, 12/01/2015

"Lecture Notes: Math 330 - Additional Material":
Ch.7.3.1: Convergence of function sequences
Ch.7.4.1-7.4.3 (First 3 appendices to ch.7) but SKIP ch.7.4.4!

Reading assignment 3: due Wed, 12/02/2015

"Lecture Notes: Math 330 - Additional Material":
Ch.7.3.2: infinite Series

Assignment 1: ONE POINT EACH for #1a, #1b, #1c.

Use thm 7.2 (Metric spaces are topological spaces), thm 3.1 (De Morgan's Law) and some propositions in ch. 7.1.5 (Convergence, contact points and closed sets) that relate open sets and closed sets to prove the following:

Theorem (Properties of closed sets in metric spaces). The following is true about closed sets of a metric space (X, d):

a.An arbitrary intersection $\bigcap_{i \in I} U_i$ of closed sets U_i is closed.

b.A finite union $U_1 \cup U_2 \cup \ldots \cup U_n$ $(n \in \mathbb{N})$ of closed sets is closed.

c.The entire set X is closed and the empty set $\emptyset$ is closed.

Assignment 2: Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$.

Hint #1: What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
Hint #2: $x^2 - 1 = (x+1)(x-1)$.
Hint #3: Only small neighborhoods matter: You may assume (but must state this reason!) that $\varepsilon < 1$ and $\delta < 1$. What kind of bounds do you get for $|\,x^2 - 1|$, $|x+1|$, $|x-1|$?
Hint #4: Put all the above together. Can you see why, for "small" δ, you obtain $|f(x) - f(x_0)| \leq 3\delta$?