Math 330 Section 1 - Spring 2016 - Homework 04

Due date: Monday, February 8, 2016 *Running total:* 18 *points*
Last submission Monday, February 22, 2016

Note: last submission was erroneously specified as 2/15/2016

Status - Reading Assignments:

Here is the status of the reading assignments you were previously asked to complete:

B/G (Beck/Geoghegan) Textbook:
all of ch.1, ch.2, ch.3

Other course material:
"Sets 1"
"Logic part 1"
"Functions part 1"

New reading assignments:

Reading assignment 1 - due: Monday, February 8:

Read carefully ch. 4.1 - 4.3 in the book. That's just a little bit more than 4 pages.

Click the link "Sets part 2" in the course materials page of the web site and read the document according to the guidelines I have outlined. Of particular importance are the three methods of proof for set identities in ch.1.11 (Set identities). It is also important that you can follow all the examples given in that document.

As is true for all the additional readings, be prepared to encounter different notation in the book, the additional material and in lecture. In lecture you will see
$A \setminus B$ instead of $A - B$ for the set difference,
$A \Delta B$ instead of $A \oplus B$ for the symmetric set difference,
Ω instead of S or U for the universal set,
$A^{\complement}$ or A^c instead of $\bar{A}$ for the complement.

Reading assignment 2 - due: Wednesday, February 10:

Read carefully the remainder of ch.4 in the book. Most of you should have encountered the binomial theorem and its applications when taking precalculus in high school. To be sure you understand the second form of induction do the following: Dig into the proof of prop.4.29 line by line and try to understand how the 2^{nd} form of induction is used in that proof.

Reading assignment 3 - due: Friday, February 12:

Click the link "Functions part 2" and read it according to the guidelines in the Additional course materials web page.

Written assignment 1:

Prove Prop. 4.6(iii) using induction: Given the definition of "Power" between props 4.5 and 4.6, prove that

$$(b^m)^k = b^{mk}$$

for $b \in \mathbb{Z}$ and $m, k \in \mathbb{Z}_{\geq 0}$.

You may use everything up to and including Prop.4.6(ii) but you won't need anything beyond the induction principle in ch.2., except for the proof of Prop.4.6(ii). That one provides an excellent template after which you can model your own proofs using induction.

Written assignment 2:

Prove Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including Prop.4.7.

Written assignment 3:

Prove Prop. 4.16(i) using induction: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \leq b < c$. Then

$$\sum_{j=a}^{c} x_j \;=\; \sum_{j=a}^{b} x_j \;+\; \sum_{j=b+1}^{c} x_j .$$

For this proof we modify the definition of Σ (B/G p.34, 35) as I discussed in class: Let $a, n \in \mathbb{Z}$ and $a \leq n$.

$$\text{(i)} \quad \sum_{j=a}^{a} x_j \;=\; x_a, \qquad\qquad \text{(ii)} \quad \sum_{j=a}^{n+1} x_j \;=\; \sum_{j=a}^{n} x_j \;+\; x_{n+1}.$$

Hints: a) Do induction on c. b) Think carefully about the base case.