Math 330 Section 1 - Spring 2016 - Homework 05

This assignment sheet was updated on 2/16 with additional hints.

Due date: February 17, 2016 *Running total:* 24 *points*
Last submission February 17, 2016(!!)
The written assignments will be GRADED ONLY ONCE!

Status - Reading Assignments:

Here is the status of the reading assignments you were previously asked to complete:

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.4

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"

New reading assignments:

Reading assignment 1 - due: Monday, February 15: Read carefully B/G ch.5.

Reading assignment 2 - due: Wednesday, February 17: Click on the latest link ("01/24/2016 version") of the "Math 330 - Additional Material" document (from now referred to as the "MF" document and read ch.2.1 - 2.5. Read carefully the subsections tagged as "Study this". Be aware that subsection 2.4.2 on families is a tough read.

Reading assignment 3 - due: Thursday, February 18: Read MF ch.2.6 and 2.7 (the remainder of ch.2).

Reading assignment 4 - due: Friday, February 19: Read carefully MF ch.3.

Written assignments: The proofs need not be as exact as doing proofs from B/G but your reasoning must be concise and without gaps. Draw some pictures to illustrate! Alltogether **those 4 assignments are worth 6 points!**

Written assignment 1:

Do exercise 2.2.1 in "Functions part 2": Let $f : \mathbb{R} \longrightarrow [0, \infty[$ be the function $x \mapsto x^2$. Is this function injective? Is it surjective? **Hint:** Be sure to first work through examples 2.2.5 and 2.2.6.

If you decide that f is NOT injective then demonstrate with a specific counterexample of two numbers that illustrate why. If you decide that f is NOT surjective then demonstrate with a specific counterexample of a number in the codomain that does not belong to the range f(domain).

Written assignment 2:

Do exercise 2.2.2 in "Functions part 2". Let $g : [0, \infty[\longrightarrow [0, \infty[$ be the function $x \mapsto x^2$. In other words, we have the same function as in assignment 1 except that we downsized its domain from $\mathbb{R}$ to $[0, \infty[$. Is this function injective? Is it surjective?

Same instructions as in the previous assignment!

Written assignment 3:

Do exercise 2.8.1 in “Functions part 2”: Find $f : A \longrightarrow B$ and $S \subseteq A$ such that $f(S^{\complement}) \neq f(S)^{\complement}$. Hint: use $f(x) = x^2$ and choose B as a one element only set (which does not leave you a whole lot of choices for A).

Written assignment 4:

Example 2.10.1 and exercise 2.10.1 in “Functions part 2” together state that
injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective

The following assignment is part of exercises 2.10.2 and 2.10.3 in “Functions part 2”.

Find functions $f : \{a\} \longrightarrow \{b_1, b_2\}$ and $g : \{b_1, b_2\} \longrightarrow \{a\}$ such that $h := g \circ f : \{a\}$ is bijective but such that it is **not true** that both f, g are injective and it is also **not true** that both f, g are surjective.

Hint: There are not a whole lot of possibilities. Draw possible candidates for f and g in arrow notation as on p.118. You should easily be able to figure out some examples. Think simple!

To get full credit, indicate clearly where injectivity or surjectivity is not obtained.