Math 330 Section 1 - Spring 2016 - Homework 07

Due date: February 22, 2016 *Running total: 30 points*
Last submission March 7, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were previously asked to complete:

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.5

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
"MF additional material", ch.2 - ch.3

New reading assignments:

Reading assignment 1 - due: Monday, February 22, 2016

Read carefully B/G ch.6.1 on equivalence relations and B/G ch.9.1 on injectivity, surjectivity and bijectivity.

Reading assignment 2 - due: Wednesday, February 24, 2016

Read carefully B/G ch.6.2: Division algorithm.

Reading assignment 3 - due: Thursday, February 25, 2016

Read carefully B/G ch.6.3: Arithmetic modulo n.

Reading assignment 4 - due: Friday, February 26, 2016

Read carefully the remainder of B/G ch.6.

Assignment 1:

Let $L : \mathbb{R}^2 \longrightarrow [0, \infty[$ be the function which assigns to a vector $\vec{x} = (x_1, x_2) \in \mathbb{R}^2$ its "length" $L(\vec{x}) := \sqrt{x_1^2 + x_2^2}$.

For two such vectors $\vec{x} = (x_1, x_2)$ and $\vec{y} = (y_1, y_2)$ we write $\vec{x} \sim \vec{y}$ iff $L(\vec{x}) = L(\vec{y})$.
a. Prove that $\sim$ is indeed an equivalence relation for $\mathbb{R}^2$.
b. Three of the following 4 points belong to the same equivalence class: $(0, 2)$, $(1, 1)$, $(2, 0)$, $(\sqrt{2}, \sqrt{2})$ Which ones?.

Assignment 2:

Let X, Y be two nonempty sets and let $f : X \longrightarrow Y$. For $a, b \in X$ we write $a \sim b$ iff $f(a) = f(b)$.
a. Prove that $\sim$ is indeed an equivalence relation for X.
b. Write $[x]_f$ for the equivalence class of $x \in X$ with respect to "$\sim$". Express $[x]_f$ in terms of the function f: $[x]_f = \{y \in X : f(y).....??.......\}$.

Assignment 3:

Remember that an equivalence relation is, like any relation for a set A, just a subset $R \subseteq A^2$ and that "$a \sim b$" really means that $(a, b) \in R$.

Prove that the empty set is an equivalence relation on the empty set.