Math 330 Section 1 - Spring 2016 - Homework 08

Due date: Monday, February 29, 2016 *Running total: 33 points*
Last submission Monday, March 14, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
"MF additional material", ch.2 - ch.3 (including the new appendices).

New reading assignments:

Reading assignment 1 - due: Monday, February 29, 2016

B/G book: Read carefully ch.7.1 and the part of ch.7.2 before thm.7.15, i.e., the algorithm for the addition of two integers. The remainder of ch.7 will be ignored.

Read carefully ch.9.1(!) in the B/G book. It contains additional material on injectivity, surjectivity and bijectivity. some of which will come in handy in MF ch.4 on cardinality which is due on Thursday.

Reading assignment 2 - due: Thursday, March 3, 2016

Read MF ch.4 (Cardinality - Alternate approach). Skip the proof of prop.4.3 ($\mathbb{N}^2$ is countable) and try to understand the subsequent remark 4.1 instead.

Reading assignment 3 - due: Friday, March 4, 2016

B/G book: Read carefully ch.8.1 - 8.3 Those chapters define axiomatically the set $\mathbb{R}$ of the real numbers (not quite true: the completeness axiom will be added in ch.8.4). Most of the material is a rehash of what you have learned in ch.1 and 2 but beware: there are new propositions dealing with the quotient of two numbers!

B/G book: Do a first reading of ch.8.4

Written assignments:

Do not use induction for any of those assignments. It would only make your task more difficult!

#1 and #2 are about proving B/G thm.6.13 (Division algorithm for integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \;=\; n \cdot q \;+\; r \quad \text{and } 0 \leq r < n.$$

Assignment 1: Uniqueness of q, r:

Prove uniqueness of the "decomposition" $m = qn + r$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$

then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Assignment 2: Existence of q, r:

Much harder than #1: Prove the existence of q and r.

Hints: Review the Well-Ordering principle from ch.2. It will give the easiest way to prove this theorem. Remember we extended the well-ordering principle as follows: Let $A \subseteq \mathbb{Z}$ have lower bounds (which is especially true if $A \subseteq \mathbb{N}$ or $A \subseteq \mathbb{Z}_{\geq 0}$). If $A \neq \emptyset$ then A has a minimum.

Apply the above to the set $A := A(m,n) := \{x \in \mathbb{Z}_{\geq 0} : x = m - kn$ for some $k \in \mathbb{Z}\}$.

Assignment 3:

Prove Thm.6.16, p.59 of B/G: Let $n \in \mathbb{Z}$. Then either n is even or $n+1$ is even.

Hint: Apply the division algorithm with $n = 2$.