Math 330 Section 1 - Spring 2016 - Homework 09

Due date: Monday, March 7, 2016, 2016 *Running total:* 36 *points*
Last submission Monday, March 21, 2016, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8.1 - ch.8.3; prelim reading of ch.8.4
all of ch.9.1

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
"MF additional material", ch.2 - ch.4

New reading assignments:

Reading assignment 1 - due: Monday, March 7, 2016

B/G book: Re-read carefully ch.8.4 and ch.9.1.
Read carefully the remainder of ch.9.

Warm-up reading for B/G chapter 10: Refresh your memory from Calc 1 or study this for the first time: Read ch.1.7: "The Precise Definition of a Limit" in Stewart 7ed. If you have a different edition of that book then a chapter of this name should exist but it may be found elsewhere (in the 6th edition this would be ch.2.4). Read that short chapter and work through its examples. The chapter contains some pictures that illustrate the meaning of the "ε–δ definition" of the limit $\lim_{x \to x_0} f(x)$ of a function $f : A \longrightarrow \mathbb{R}$ with domain $A \subseteq \mathbb{R}$ at $x = x_0$. Be sure you understand how those pictures relate to the definition. **It is crucial that you understand this definition and can cite it from memory** – not literally of course, but such that what you write coincides mathematically with what's in the book.

We skip B/G ch.10, 11, 12 for now. You should have knowledge of each of the following facts which are dealt with in those chapters:

a.	Rational numbers are fractions or periodic decimals.
b.	There are real numbers which are not rational. Examples: $\sqrt{2}, \pi$.
c.	All real numbers can be written as decimals.

Reading assignment 2 - due: Wednesday, March 9, 2016

Read carefully B/G ch.13.1 and 13.2 **without looking at the proofs**.

Reading assignment 3 - due: Thursday, March 3, 2016

Reread carefully MF ch.4. I shall present cardinality in class according to that chapter.

Reading assignment 4 - due: Friday, March 4, 2016

Read carefully B/G ch.13.3 and 13.4 **including proofs**.

Assignment 1:

Prove Prop.7.1 using induction: If $n \in \mathbb{N}$ then $n < 10^n$. You may use the fact that 10 (defined as $9+1$) satisfies $0 < 1 < 2 < 10$. Justify your inequalities referring to prop. 2.7(i) - 2.7(iv).

Assignment 2:

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A := A(n) := \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) := \min(A).$$

Prop.7.3 states that, for all $n \in \mathbb{N},\ \nu(n) = k \iff 10^{k-1} \leqq n < 10^k$.

Prove "$\Rightarrow$" of prop.7.3.

Assignment 3: Prove "$\Leftarrow$" of prop.7.3.

The math for assignments 2 and 3 is easy but you may find it hard to write down a proof that meets my demands for precision.

Hints for #2 and #3: 1) I gave the set a name (A) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** A: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)