Math 330 Section 1 - Spring 2016 - Homework 11

Due date: March 21, 2016 *Running total:* 41 *points*
Last submission April 8, 2016 (Fri!)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8 - ch.9
ch.10.1-10.3.
ch.13.1 and 13.2 **without looking at the proofs**
ch.13.3 and 13.4 **including proofs**

"MF additional material":
ch.2 - ch.4
ch.5 up to and including def.5.12 (Tail sets of a sequence): ch.6
ch.7 until before def.7.10 "Basis and neighborhood basis" in ch.7,1,3
ch.7.1.4 and 7.1.5

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due: Monday, March 21 Read B/G ch.13.5 (nondescribable numbers: this was handled in lecture on Fri, March 18).

Reread B/G ch.10.1-10.3

Reading assignment 2 - due: Wednesday, March 23

Read carefully B/G ch.10.4-10.5 and reread Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit".

Reading assignment 3 - due: Thursday, March 24

Reread MF ch.6 (just for understanding).
Reread carefully the material of MF ch.7 that was already assigned.

OPTIONAL Reading assignment 4 - due: Friday, March 25

On Friday I shall talk about an alternate approach of construction the real numbers. The methods used will be those used in B/G project 6.9 (which I talked about in class) and in B/G prop.6.25, 6.26. You will find it easier to follow that lecture if you take another look how one can extend a set with algebraic operations such as $(\mathbb{Z}, +, \cdot)$ to a larger one such as the rational numbers by forming equivalence classes and extending the operations to those classes.

Assignment 1:

Prove B/G Prop.13.3: Let $k, n \in \mathbb{N}$ such that $1 \leq k < n$. Then the function

$$g_k : [n-1] \longrightarrow [n] \setminus \{k\} \quad \text{defined by} \quad g_k(j) := \begin{cases} j & \text{if } j < k \\ j+1 & \text{if } j \geq k \end{cases} \tag{0.1}$$

is bijective. **Hint:** Computing the inverse might be easiest, but be sure to **prove** that both $g_k \circ g_k^{-1} = id_{[n] \setminus \{k\}}$ and $g_k^{-1} \circ g_k = id_{[n-1]}$!

Assignment 2:

Prove B/G Cor.13.18: $\mathbb{Q}$ is countable. You may use all of B/G ch.13 up to thm.13.12 **plus** thm.13.19 (a countable union of countable sets is countable).
Hint: For $n \in \mathbb{N}$ let $Q_n := \{m/n : m \in \mathbb{Z} \text{ and } -n^2 \leq m \leq n^2\}$. They might come in handy!