Math 330 Section 1 - Spring 2016 - Homework 12

Due date: April 4, 2016 *Running total:* 44 *points*
Last submission April 20, 2016

***Updated on April 13, 2016**: A massive hint has been given for solving assignment 1. The last submission date was moved from April 18 to April 20.*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8 - ch.10
ch.13.1 and 13.2 **without looking at the proofs**
ch.13.3, 13.4 and 13.5 **including proofs**

"MF additional material":
ch.2 - ch.4
ch.5 up to and including def.5.12 (Tail sets of a sequence): ch.6
ch.7 until before def.7.10 "Basis and neighborhood basis" in ch.7,1,3
ch.7.1.4 and 7.1.5

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due: Wednesday, April 6 Read carefully B/G ch.11.

Reading assignment 2 - due: Friday, April 8 Finish MF ch.5 as follows: Skim through the material from prop.5.3 through prop.5.5 but read everything else carefully. Memorize thm 5.1 and 5.2.

Assignment 1:

Prove B/G prop.10.10(iv): $x, y \in \mathbb{R} \Rightarrow |x-y| \geq \big||x|-|y|\big|$.

To show this use the following proposition:

Proposition. Let $a, b \in \mathbb{R}$ such that both #1) $-a \leqq b$ and #2) $a \leqq b$. Then $|a| \leqq b$.

Proof of proposition: Case 1) $a \geqq 0$: It follows from #2 that $|a| = a \leqq b$ which is what we had to show.
Case 2) $a < 0$: It follows from #1 that $|a| = -a \leqq b$ which is what we had to show. ■.

Hint f. assignment 1: first use the triangle inequality on $|x| = |(x-y)+y|$ and then on $|y| = |(y-x)+x|$. See what you get for $a := |x|-|y|$ and $b := |x-y|$.

Assignment 2:

Prove B/G prop.10.16: $\lim_{k\to\infty} x_k = L \;\Rightarrow\; \lim_{k\to\infty} x_{k+1} = L$.
Hint: For a clean proof define $y_k := x_{k+1}$.

Assignment 3:

Prove B/G prop.10.21(ii): Let $\lim_{k\to\infty} x_k = L$. If $(x_k)_{k=0}^{\infty}$ is decreasing then $x_k \geq L$ for all $k \geq 0$.