Math 330 Section 1 - Spring 2016 - Homework 14

Due date: April 22, 2016 *Running total:* 51 *points*
Last submission May 6, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8 - ch.12
ch.13.1 and 13.2 **without looking at the proofs**
ch.13.3, 13.4 and 13.5 **including proofs**

"MF additional material":
ch.2 - ch.6 (prop.5.3 through prop.5.5 without proofs)
ch.6
all of ch.7.1 but see this footnote. [1] ch.7.2, until the end of ch.7.2.2.

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due: Friday, April 22 Read carefully the remainder of MF ch.7.2.

Reading assignment 2 - due: Monday, April 25 Read carefully MF ch.7.3.

Assignment 1:

Let $A \subseteq (E, d)$. Definition 7.13 (Metric subspaces) states that $(A, d\big|_{A\times A})$ is a metric space in the sense of def.7.1. Prove it!

Assignment 2:

Given is a non-empty set E. Find the smallest possible subset $\mathfrak{U}$ of 2^E that defines a topology on E.

Assignment 3: (TWO POINTS!)

Let (E, d) be N–dimensional space $\mathbb{R}^N$ with the Euclidean metric $d(\vec{x}, \vec{y}) := \|\vec{x} - \vec{y}\|_2 = \sqrt{\sum_{j=1}^{N} (x_j - y_j)^2}$.

Prove that the closure of the subset $\mathbb{Q}^N$ of all vectors with rational coordinates is all of $\mathbb{R}^N$.
Hints: **a.** Remember that for any two real numbers $x < y$ there exists rational r such that $x < r < y$ (B/G thm.11.8). **b.** Let $\varepsilon > 0$. Let $\vec{x} = (x_1, \dots, x_N)$ and $\vec{y} = (y_1, \dots, y_N)$. such that $|x_j - y_j| < \varepsilon/N$ for all j. Show that $d(\vec{x}, \vec{y}) < \varepsilon$. **c.** Conclude from parts **a** and **b** that any ε–neighborhood of $\vec{x}$ contains an element of $\mathbb{Q}^N$ and

[1] Ch.7.1.3: Read carefully only until def.7.10 (Basis and neighborhood basis) and skim through the remainder.

finish the proof.

You will get one point for showing **b** and another point for finishing the proof.