Math 330 Section 1 - Spring 2016 - Homework 15

Due date: Wed, April 27, 2016 *Running total: nn points*
Last submission Monday, May 9, 2016

***Updated on 2016-04-28**: a hint was given to assignment 1.*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8 - ch.12
ch.13.1 and 13.2 **without looking at the proofs**
ch.13.3, 13.4 and 13.5 **including proofs**

"MF additional material":
ch.2 - ch.6 (prop.5.3 through prop.5.5 without proofs)
ch.6
all of ch.7.1 - 7.3 but see this footnote. [1]

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due: Wednesday, April 27 Read carefully ch.7.4: Addenda to ch.7 (Convergence and continuity). This contains a single proposition and proof: uniform convergence of functions is convergence with respect to the sup norm.

Read carefully B/G appendix A "Continuity and Uniform Continuity"

Reading assignment 2 - due: Friday, April 29 Read carefully MF ch.8 until the end of ch.8.3 (ε-nets and total boundedness). Note that you can skip the proof of prop.8.2 which constitutes the major part of ch.8.3.

Assignment 1:

MF Prop.7.2 (Metric properties of the distance between real functions):
Let X be an arbitrary, non–empty set.
Let $\mathscr{B}(X,\mathbb{R}) := \{h(\cdot) : h(\cdot) \text{ is a bounded real function on } X\}$.
Let $f(\cdot), g(\cdot), h(\cdot) \in \mathscr{B}(X,\mathbb{R})$ Then the distance function

$$d(\cdot) : \ \mathscr{B}(X,\mathbb{R}) \times \mathscr{B}(X,\mathbb{R}) \longrightarrow \mathbb{R}_+ \qquad (h_1,h_2) \longmapsto d(h_1,h_2) := \|h_1 - h_2\|_\infty$$

[1] Ch.7.1.3: Read carefully only until def.7.10 (Basis and neighborhood basis) and skim through the remainder of ch.7.1.3.

satisfies the three properties of a metric:

(0.1a) $d(f,g) \geqq 0 \quad \forall f(\cdot), g(\cdot) \in \mathscr{B}(X,\mathbb{R}) \quad \text{and} \quad d(f,g) = 0 \iff f(\cdot) = g(\cdot)$ **positive definite**

(0.1b) $d(f,g) = d(g,f) \quad \forall f(\cdot), g(\cdot) \in \mathscr{B}(X,\mathbb{R})$ **symmetry**

(0.1c) $d(f,h) \leqq d(f,g) + d(g,h) \quad \forall\, f,g,h \in \mathscr{B}(X,\mathbb{R})$ **triangle inequality**

Hint for the proof of the triangle inequality: Use prop. 5.9 in ch.5.4 (Appendix: Addenda to chapter 5). Apply it with $\varphi(x) := f(x) - g(x)$, etc. I won't allow this unless you reference that proposition in your homework!

Assignment 2:

Let $A := \{(x_1, x_2) \in \mathbb{R}^2 : x_1 > 0, x_2 > 0\}$ be the first quadrant in the plane (the points on the coordinate axes are excluded). Prove that each element of A is an inner point, i.e., A is open in $\mathbb{R}^2$.