Math 330 Section 1 - Spring 2016 - Homework 16

Due date: Mon, May 2, 2016 *Running total:* 55 *points*
Last submission Wednesday, May 11, 2016(!!)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 - ch.6
all of ch.7.1; ch.7.2 until before thm.7.15
all of ch.8 - ch.12
ch.13.1 and 13.2 **without looking at the proofs**
ch.13.3, 13.4 and 13.5 **including proofs**
Appendix A "Continuity and Uniform Continuity"

"MF additional material":
ch.2 - ch.6 (prop.5.3 through prop.5.5 without proofs)
ch.6
all of ch.7 but see this footnote [1] .
ch.8.1 - 8.3, omit the proof of prop.8.2.

Other course material:
"Logic part 1"
"Sets part 1", "Sets part 2",
"Functions part 1", "Functions part 2"
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit"

New reading assignments:

Reading assignment 1 - due: Monday, May 2 Review MF ch.8.1 - 8.3, especially the definition of compactness that uses finite subcovers of open sets.

Reading assignment 2 - due: Wednesday, May 4 Read carefully MF ch.8.4: Sequence compactness

Reading assignment 3 - due: Thursday, May 5
a. Read carefully MF ch.8.5: Continuous functions and compact spaces.
b. Read carefully MF ch.8.6: Heine Borel theorem but skip ch.8.6.1.

Reading assignment 4 - due: Friday, May 6
a. Read MF ch.9.1.1 (Partially ordered sets). Be sure to learn Zorn's lemma.
b. Read MF ch.9.1.2 (Partially ordered sets). up to and including lemma 9.2.

Assignment 1:

MF Proposition 7.12 does NOT have a proof of the following: Let (X, d) be a metric space and $f, g : X \longrightarrow \mathbb{R}$ both be continuous at $x_0 \in X$. Prove that if $g(x_0) \neq 0$ then f/g is continuous at x_0 in two stages.

a. prove that $h(x) := 1/g(x)$ is continuous at x_0. **Hint:** Use sequence continuity and results in B/G ch.10.4 concerning sequences of real numbers. Don't attempt to use the $\varepsilon - \delta$ characterization of continuity.

[1] Ch.7.1.3: Read carefully only until def.7.10 (Basis and neighborhood basis) and skim through the remainder of ch.7.1.3.

b. Now rewrite f/g as a ________ and use a result in MF ch.7.2.2 to conclude that f/g is continuous at x_0

Assignment 2:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$.

Hint #1: What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
Hint #2: $x^2 - 1 = (x + 1)(x - 1)$.
Hint #3: Only small neighborhoods matter: You may assume (but must state this reason!) that $\varepsilon < 1$ and $\delta < 1$. What kind of bounds do you get for $|\, x^2 - 1|$, $|x + 1|$, $|x - 1|$?
Hint #4: Put all the above together. Can you see why, for "small" δ, you obtain $|f(x) - f(x_0)| \leq 3\delta$?