Math 330 Section 1 - Fall 2016 - Homework 04

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.3

MF lecture notes:
ch.1, ch.2, ch.4 until before ch.4.2.2 (function def.)

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1, section 1

New reading assignments:

Reading assignment 1 - due Monday, September 12:
Read carefully the remainder of MF ch.4. This is important as it covers the definition of a function!
Read carefully B/G ch.5.
Optional (for examples): Ch.4.1: (Set Ops) and Ch.4.2: Properties of Functions of B/K

That's a lot of pages but the MF document is mostly about examples.

Reading assignment 2 - due: Tuesday, September 13:
MF doc: Read carefully ch.5.1 and read the remainder of ch.5.

Reading assignment 3 - due Wednesday, September 14:
MF doc: Read carefully ch.6. It is very brief but extremely important and rather terse.

Reading assignment 4 - due Friday, September 16:
Read carefully the remainder of ch.1 in B/G.
B/G: Read carefully ch.4.1-4.4.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

Prove B/G Prop. 4.6(iii) using induction: Given the definition of "Power" between props 4.5 and 4.6, prove that

$$(b^m)^k = b^{mk}$$

for $b \in \mathbb{Z}$ and $m, k \in \mathbb{Z}_{\geq 0}$.

You may use everything up to and including Prop.4.6(ii) but you won't need anything beyond the induction principle in ch.2., except for the proof of Prop.4.6(ii). That one provides an excellent template after which you can model your own proofs using induction.

Written assignment 2:

Prove B/G Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including Prop.4.7.

Written assignment 3:

Prove B/G Prop. 4.16(i) using induction: Let $(x_j)_{j\in\mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \le b < c$. Then

$$\sum_{j=a}^{c} x_j = \sum_{j=a}^{b} x_j + \sum_{j=b+1}^{c} x_j .$$

For this proof we modify the definition of Σ (B/G p.34, 35) as I discussed in class: Let $a, n \in \mathbb{Z}$ and $a \le n$.

$$\text{(i)} \quad \sum_{j=a}^{a} x_j = x_a, \qquad \text{(ii)} \quad \sum_{j=a}^{n+1} x_j = \sum_{j=a}^{n} x_j + x_{n+1}.$$

Hints: a) Do induction on c. b) Think carefully about the base case.