Math 330 Section 1 - Fall 2016 - Homework 05

Published: Saturday, September 17, 2016 *Running total: 26 points*
Last submission: Friday, September 23, 2016 ***NO RESUBMISSIONS***

This homework is worth 8 points!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.3, ch.4.1-4.4, ch.5

MF lecture notes:
ch.1, ch.2, ch.4-ch.6

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1.1, ch.4.1, ch.4.2

New reading assignments: None: They will come with homework 6.

Written assignment 1:

Injectivity and Surjectivity

- Let $f : \mathbb{R} \longrightarrow [0, \infty[; \quad x \mapsto x^2$.
- Let $g : [0, \infty[\longrightarrow [0, \infty[; \quad x \mapsto x^2$.
  In other words, g is same function as f as far as assigning function values is concerned, but that its domain was downsized to $[0, \infty[$.

Answer the following with **true** or **false**.

a. f is surjective **c.** g is surjective
b. f is injective **d.** g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2:

Let $A \subseteq \mathbb{R}$.

- Let $F_1 : A \longrightarrow [-2, 20[; \quad x \mapsto x^2$.
- Let $F_2 : A \longrightarrow [2, 20[; \quad x \mapsto x^2$.
- Let $G_1 : A \longrightarrow [-2, 20[; \quad x \mapsto \sqrt{x}$.
- Let $G_2 : A \longrightarrow [2, 20[; \quad x \mapsto \sqrt{x}$.
- Let $G_3 : A \longrightarrow [-20, 2[; \quad x \mapsto \sqrt{x}$.
- Let $G_4 : A \longrightarrow [-20, -2[; \quad x \mapsto \sqrt{x}$.

What choice of A makes

a. F_1 surjective? **c.** F_2 surjective? **e.** G_1 surjective? **g.** G_2 surjective?
b. F_1 injective? **d.** F_2 injective? **f.** G_1 injective? **h.** G_2 injective?

i. G_3 surjective? **k.** G_4 surjective?
j. G_3 injective? **l.** G_4 injective?

For the questions above

- Write **impossible** if no choice of $A \subseteq \mathbb{R}$ exists.
- Write **NAF** for any of $F_1, F_2, G_1, G_2, G_3, G_4$ which does **not define a function**.

Written assignment 3:

Find $f : X \longrightarrow Y$ and $A \subseteq X$ such that $f(A^{\complement}) \neq f(A)^{\complement}$. Hint: use $f(x) = x^2$ and choose Y as a **one element only** set (which does not leave you a whole lot of choices for X). See example 4.17 on p.78.

Written assignment 4:

You will learn later in this course that
injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective.

The following illustrates that the reverse is not necessarily true.

Find functions $f : \{a\} \longrightarrow \{b_1, b_2\}$ and $g : \{b_1, b_2\} \longrightarrow \{a\}$ such that $h := g \circ f : \{a\}$ is bijective but such that it is **not true** that both f, g are injective and it is also **not true** that both f, g are surjective.

Hint: There are not a whole lot of possibilities. Draw possible candidates for f and g in arrow notation as on p.118. You should easily be able to figure out some examples. Again, think simple and look at example 4.17 on p.78.