Math 330 Section 1 - Fall 2016 - Homework 07

Published: Saturday, September 24, 2016 *Running total:* 36 *points*
Last submission: Friday, October 7, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.6

MF lecture notes:
ch.1, ch.2, ch.4-ch.6

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1.1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, September 26:
Reread the last page or so of B/G ch.2 about gcd$(m, n) := \min(S)$.
Read carefully B/G ch.7.1 and **skim** ch.7.2. (base 10 arithmetic).
Read carefully B/G ch.8.1.

Reading assignment 2 - due: Tuesday, September 27:
Read carefully B/G ch.8.2 and **compare** ch.8.1 and ch.8.2 to the results for $\mathbb{Z}$ in B/G ch.1 and ch.2.

Reading assignment 3 - due Wednesday, September 28:
Read carefully the remainder of B/G ch.8.

Reading assignment 4 - due Friday, September 30:
Read carefully B/G ch.9.1 about injectivity and surjectivity.

Written assignment 1: (One point each for **a** and **b**)

Prove formula b of De Morgan's Law: Let there be a universal set Ω.
Then for any indexed family $(A_\alpha)_{\alpha\in I}$ of sets:

$$\textbf{b.}\ \Big(\bigcap_\alpha A_\alpha\Big)^{\complement} = \bigcup_\alpha A_\alpha^{\complement}$$

a: Prove "$\subseteq$". **b:** Prove "$\supseteq$".

Written assignment 2: (One point each for **a** and **b**)

Let $L : \mathbb{R}^2 \longrightarrow [0, \infty[$ be the function which assigns to a vector $\vec{x} = (x_1, x_2) \in \mathbb{R}^2$ its "length" $L(\vec{x}) := \sqrt{x_1^2 + x_2^2}$.

For two such vectors $\vec{x} = (x_1, x_2)$ and $\vec{y} = (y_1, y_2)$ we write $\vec{x} \sim \vec{y}$ iff $L(\vec{x}) = L(\vec{y})$.

- **a:** Prove that $\sim$ is indeed an equivalence relation for $\mathbb{R}^2$.
- **b:** Three of the following 4 points belong to the same equivalence class:
  $(0, 2)$, $(1, 1)$, $(2, 0)$, $(\sqrt{2}, \sqrt{2})$ Which ones?.

Written assignment 3: (One point each for **a** and **b**)

Let X, Y be two nonempty sets and let $f : X \longrightarrow Y$. For $a, b \in X$ we write $a \sim b$ iff $f(a) = f(b)$.

a: Prove that $\sim$ is indeed an equivalence relation for X.

b: Write $[x]_f$ for the equivalence class of $x \in X$ with respect to "$\sim$". Express $[x]_f$ in terms of the function f: $[x]_f = \{x' \in X : f(x')$.....??.......$\}$. (I do not want to see "$[x]_f = \{x' \in X : x' \sim x\}$".)