Math 330 Section 1 - Fall 2016 - Homework 08

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.8 (skim 7.2)
ch.9.1

MF lecture notes:
ch.1, ch.2, ch.4-ch.6

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1.1, ch.4.1, ch.4.2

New reading assignments:

No new reading assignments but use the time to reread the old stuff!

Written assignments:

Do not use induction for any of those assignments. It would only make your task more difficult!

#1 and #2 are about proving B/G thm.6.13 (Division algorithm for integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m = n \cdot q + r \quad \text{and } 0 \leq r < n.$$

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints: Review the Well-Ordering principle from ch.2. It will give the easiest way to prove this theorem. Remember we extended the well-ordering principle as follows: Let $A \subseteq \mathbb{Z}$ have lower bounds (which is especially true if $A \subseteq \mathbb{N}$ or $A \subseteq \mathbb{Z}_{\geq 0}$). If $A \neq \emptyset$ then A has a minimum.

Apply the above to the set $A := A(m, n) := \{x \in \mathbb{Z}_{\geq 0} : x = m - kn \text{ for some } k \in \mathbb{Z}\}$.

Written assignment 3:

Prove Thm.6.16, p.59 of B/G: Let $m \in \mathbb{Z}$. Then either m is even or $m + 1$ is even.

Hint: Apply the division algorithm with $n = 2$.