Math 330 Section 1 - Fall 2016 - Homework 09

Published: Saturday, October 8, 2016 *Running total:* 42 *points*
Last submission: Friday, October 21, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.8 (skim 7.2)
ch.9.1

MF lecture notes:
ch.1, ch.2, ch.4-ch.6

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1.1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, October 10:
Read carefully B/G ch.9.2
Read carefully B/G ch.10 until the start of ch.10.4 (Limits)

Reading assignment 2 - due: Tuesday, October 11:
Read carefully ch.10.4 of B/G through prop. 10.22.

Reading assignment 3 - due Friday, October 14:
Read carefully the remainder of B/G ch.10.

Written assignment 1:

Prove B/G Prop.7.1 using induction: If $n \in \mathbb{N}$ then $n < 10^n$. You may use the fact that 10 (defined as $9+1$) satisfies $0 < 1 < 2 < 10$. Justify your inequalities referring to B/G prop. 2.7(i) - 2.7(iv).

Written assignment 2:

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A \ := A(n) \ := \ \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) \ := \ \min(A).$$

B/G prop.7.3 states that, for all $n \in \mathbb{N}$, $\nu(n) = k \iff 10^{k-1} \leqq n < 10^k$.

Prove "$\Rightarrow$" of B/G prop.7.3.

Written assignment 3:

Prove "$\Leftarrow$" of B/G prop.7.3.

The math for assignments 2 and 3 is easy but you may find it hard to write down a proof that meets my demands for precision.

Hints for #2 and #3: 1) I gave the set a name (A) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** A: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)