Math 330 Section 1 - Fall 2016 - Homework 15

Published: Friday, November 18, 2016 *Running total:* 59 *points*
Last submission: Friday, December 2, 2016

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
ch.1 - ch.12 (skim 7.2)
Appendix A: Continuity & Uniform Continuity

MF lecture notes:
ch.1, ch.2, ch.4-ch.6, ch.8-ch.10

B/K lecture notes (optional reading – good for examples, improved understanding):
ch.1.1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, November 21:
Reread MF ch.5.3 (Countable sets)
Read carefully B/G ch.13 (Cardinality)

Reading assignment 2 - due: Tuesday, November 22:
Read carefully MF ch.7 (Cardinality)

Reading assignment 3 - OPTIONAL - due Wednesday, November 23:
Read MF ch.14: The better you are prepared, the more you will get out of that optional lecture.
Also recommended: take a look at B/G prop.6.25, 6.26 (constructing $\mathbb{Z}_n$ from $\mathbb{Z}$ via equivalence classes). You will find it easier to follow that lecture if you get a feeling for how one can take a set with algebraic operations such as $(\mathbb{Z}, +, \cdot)$ and create from it a new one by forming equivalence classes and adapting the operations to those classes.

Reading assignment 4 - due Monday(!!), November 28:
Read carefully MF ch.11 up to and including ch.11.4, prop 11.4
(Sequence compact implies completeness)

Written assignment 1: Prove B/G Thm.11.12, p.110: If $r \in \mathbb{N}$ is not a perfect square, then $\sqrt{r}$ is irrational.
Hint: Study the proof of prop.11.10 carefully and you'll see that you can use it with small alterations.

Written assignment 2: Use everything up-to and including B/G prop.11.10 PLUS all of B/G prop.11.20 and B/G prop.11.21 to prove the following: Let $m, n \in \mathbb{Z} \setminus \{0\}$. Then $(m/n)\sqrt{2}$ is irrational.

Written assignment 3: Prove B/G Prop.13.3: Let $k, n \in \mathbb{N}$ such that $1 \leq k < n$. Then the function

$$g_k : [n-1] \longrightarrow [n] \setminus \{k\} \quad \text{defined by} \quad g_k(j) := \begin{cases} j & \text{if } j < k \\ j+1 & \text{if } j \geq k \end{cases}$$

is bijective. **Hint:** Computing the inverse might be easiest, but be sure to **prove** that both $g_k \circ g_k^{-1} = id_{[n]\setminus\{k\}}$ and $g_k^{-1} \circ g_k = id_{[n-1]}$!