Math 330 Section 2 - Spring 2017 - Homework 07

Published: Thursday, February 9, 2017 Running total: 34 points

Last submission: Friday, February 24, 2017

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:

• all of ch.1 - ch.5

MF lecture notes:

- ch.1 ch.2, ch.4 ch.6
- **b.** ch.16 (addenda to B/G text)

B/K lecture notes (optional reading – good for examples, improved understanding):

• ch.1 – section 1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, February 13:

• Read carefully B/G ch.6.1 - 6.3.

Reading assignment 2 - due: Tuesday, February 14:

• Read carefully the remainder of B/G ch.6.

Reading assignment 3 - due Wednesday, February 15:

• Read carefully B/G ch.7 until before prop.7.10.

Reading assignment 4 - due Friday, February 17:

• Read the remainder of B/G ch.7. Read it carefully until before thm.7.17.

Written assignment 1: (One point each for a and b)

Prove formula b of De Morgan's Law: Let there be a universal set Ω . Then for any indexed family $(A_{\alpha})_{\alpha \in I}$ of sets:

b.
$$\left(\bigcap_{\alpha} A_{\alpha}\right)^{\complement} = \bigcup_{\alpha} A_{\alpha}^{\complement}$$

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a: Prove " \subseteq ". **b:** Prove " \supseteq ".

Written assignment 2: (One point each for a and b)

Let $L: \mathbb{R}^2 \longrightarrow [0, \infty[$ be the function which assigns to a vector $\vec{x} = (x_1, x_2) \in \mathbb{R}^2$ its "length" $L(\vec{x}) := \sqrt{x_1^2 + x_2^2}$.

For two such vectors $\vec{x} = (x_1, x_2)$ and $\vec{y} = (y_1, y_2)$ we write $\vec{x} \sim \vec{y}$ iff $L(\vec{x}) = L(\vec{y})$.

- **a:** Prove that \sim is indeed an equivalence relation for \mathbb{R}^2 .
- **b:** Three of the following 4 points belong to the same equivalence class: (0,2), (1,1), (2,0), $(\sqrt{2},\sqrt{2})$ Which ones?.

Written assignment 3: (One point each for a and b)

Let X, Y be two nonempty sets and let $f: X \longrightarrow Y$. For $a, b \in X$ we write $a \sim b$ iff f(a) = f(b).

- **a:** Prove that \sim is indeed an equivalence relation for X.
- **b:** Write $[x]_f$ for the equivalence class of $x \in X$ with respect to " \sim ". Express $[x]_f$ in terms of the function $f: [x]_f = \{x' \in X : f(x'), \dots, ??, \dots\}$. (I do not want to see " $[x]_f = \{x' \in X : x' \sim x\}$ ".)