Math 330 Section 2 - Spring 2017 - Homework 08

Published: Friday, February 17, 2017 *Running total: 37 points*
Last submission: Wednesday, March 8, 2017

***Note**: Wednesday, March 8 is the first day after winter break.*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 - ch.7; (ch.7 carefully until before thm.7.17)

MF lecture notes:
- ch.1 - ch.2, ch.4 - ch.6
- ch.16 (addenda to B/G text)

B/K lecture notes (optional reading – good for examples, improved understanding):
- ch.1 – section 1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, February 20:
- Read carefully B/G ch.8.1 - ch.8.3.

Reading assignment 2 - due: Tuesday, February 21:
- Compare B/G ch.8.1 and ch.8.2 to the results for $\mathbb{Z}$ in B/G ch.1 and ch.2

Reading assignment 3 - due Wednesday, February 22:
- Read carefully the remainder of B/G ch.8.

Reading assignment 4 - due Friday, February 24:
- Read carefully B/G ch.9.1. You have basically seen all of it in MF ch.4.

Written assignments:

Do not use induction for any of those assignments. It would only make your task more difficult!

#1 and #2 are about proving B/G thm.6.13 (Division algorithm for integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m \,=\, n \cdot q \,+\, r \quad \text{and } 0 \leq r < n.$$

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints: Review the Well-Ordering principle from ch.2. It will give the easiest way to prove this theorem. Remember we extended the well-ordering principle as follows: Let $A \subseteq \mathbb{Z}$ have lower bounds (which is especially true if $A \subseteq \mathbb{N}$ or $A \subseteq \mathbb{Z}_{\geq 0}$). If $A \neq \emptyset$ then A has a minimum.

Apply the above to the set $A := A(m,n) := \{x \in \mathbb{Z}_{\geq 0} : x = m - kn$ for some $k \in \mathbb{Z}\}$.

Written assignment 3:

Prove Thm.6.16, p.59 of B/G: Let $m \in \mathbb{Z}$. Then either m is even or $m+1$ is even.

Hint: Apply the division algorithm with $n = 2$.