Math 330 Section 2 - Spring 2017 - Homework 09

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 - ch.8 (ch.7 carefully until before thm.7.17)
- ch.9.1

MF lecture notes:
- ch.1 - ch.2, ch.4 - ch.6
- ch.16 (addenda to B/G text)

B/K lecture notes (optional reading – good for examples, improved understanding):
- ch.1 – section 1, ch.4.1, ch.4.2

New reading assignments:

Reading assignment 1 - due Monday, February 27:
- Read carefully MF ch.8.1 (Minima, Maxima, Infima and Suprema) up to and including prop.8.3, i.e., you stop before the definition of the convergence of sequences. Be sure you understand the difference between maxima and suprema!

Reading assignment 2 - due: Tuesday, February 28:
- Read carefully the remainder of B/G ch.9.

Reading assignment 3 - due Wednesday, March 1:
- Read carefully the B/G ch.10.1 and 10.2.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

Prove B/G Prop.7.1 using induction: If $n \in \mathbb{N}$ then $n < 10^n$. You may use the fact that 10 (defined as $9+1$) satisfies $0 < 1 < 2 < 10$. Justify your inequalities referring to B/G prop. 2.7(i) - 2.7(iv).

Written assignment 2:

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A \ := A(n) \ := \ \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) \ := \ \min(A).$$

B/G prop.7.3 states that, for all $n \in \mathbb{N},\ \nu(n) = k \iff 10^{k-1} \leqq n < 10^k$.

Prove "⇒" of B/G prop.7.3.

Written assignment 3:

Prove "$\Leftarrow$" of B/G prop.7.3.

The math for assignments 2 and 3 is easy but you may find it hard to write down a proof that meets my demands for precision.

Hints for #2 and #3: 1) I gave the set a name (A) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** A: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)