Math 330 Section 2 - Spring 2017 - Homework 11

Published: Friday, March 10, 2017 *Running total:* 43 *points*
Last submission: March, February 24, 2017

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 - ch.10 (ch.7 carefully until before thm.7.17)

MF lecture notes:
- ch.1 - ch.2, ch.4 - ch.6
- ch.8.1 up to and including prop.8.3
- ch.16 (addenda to B/G text)

Other material:
- B/K lecture notes ch.1 – section 1, ch.4.1, ch.4.2
  (optional reading – good for examples, improved understanding)
- Stewart Calculus: "The Precise Definition of a Limit" (ch.1.7 in the 7th edition).

New reading assignments:

Reading assignment 1 - due Monday, March 13:
- **a.** Read carefully B/G ch.11 up to and including cor.11.23. Skip the remainder.
- **b.** Read carefully B/G ch.12.1. Think back to what you learned about series when taking calculus.

Reading assignment 2 - due Tuesday, March 14:
- **a.** Read carefully the remainder of B/G ch.12.

Reading assignment 3 - due Wednesday, March 15:
- **a.** Continue MF ch.8.1 until before prop.8.8. Read this carefully!
- **b.** While reading about liminf and limsup, look at what happens for the sequences
  $x_n := (-1)^n$, $y_n := (-1)^n\big(1 + \frac{1}{n}\big)$ (both not convergent) and $z_n := 1 + \frac{1}{n}$ (converges to 1).

Reading assignment 4 - due: Friday, March 17:
- **a.** Continue MF ch.8.1 until before thm.8.1. **Skip all proofs** but draw some pictures
- **b.** Read carefully the remainder of MF ch.8.1.

Written assignment 1:
Let $x, y \in \mathbb{R}$ such that $x < y$. Let $z := (x+y)/2$. Prove that $x < z < y$.
Hint: Prove first that $2x < x + y < 2y$. Then use B/G prop.8.37(ii): $\big[\alpha > 0$ and $\alpha u < \alpha v \;\Rightarrow\; u < v\big]$
to show that $x < z < y$. Be sure to indicate explicitly your choice of α!

Written assignment 2:
Prove the following part of B/G prop.8.49:
Let $A \subseteq \mathbb{R}$ such that $A \neq \emptyset$. If $\sup(A)$ exists and if $\sup(A) \in A$ then $\max(A)$ exists and $\max(A) = \sup(A)$.

Written assignment 3:
Use everything up to and including B/G prop.8.45 to prove B/G prop.8.47: $\mathbb{R}_{>0}$ has no upper bounds.
Hint: 1) First prove that if $x \notin \mathbb{R}_{>0}, i.e., x = 0$ or $x < 0$, then x is not an upper bound of $\mathbb{R}_{>0}$. 2) Then prove that if $x \in \mathbb{R}_{>0}$ then x is not an upper bound of $\mathbb{R}_{>0}$.