Math 330 Section 2 - Spring 2017 - Homework 15

Published: Friday, February 3, 2017 *Running total: 55 points*
Last submission: April 18, 2017

This assignment is worth 5 points. It is due BEFORE #14 and graded only ONCE.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 - ch.13 (ch.7 carefully until before thm.7.17, ch.11 until cor.11.23)

MF lecture notes:
- ch.1 - ch.2, ch.4 - ch.9 (skipped the proof of prop.7.3)
- MF ch.10 until before ch.10.2.5 (uniform continuity).
- ch.16 (addenda to B/G text)

Other material:
- B/K lecture notes ch.1 – section 1, ch.4.1, ch.4.2
  (optional reading – good for examples, improved understanding)
- Stewart Calculus: "The Precise Definition of a Limit" (ch.1.7 in the 7th edition).

Note on the reading assignments:

- I spend a lot of time planning those assignments. Sometimes it takes me more than 45 minutes, especially when intermingling material from both the B/G text and the MF document. The reading for an entire week is broken down into separate pieces, one for each lecture, so that the work load is evenly distributed.
- I stated at the beginning of the semester that everything that was assigned for reading until day X is fair game for a quiz given at day X, whether it was announced or not and regardless of whether the material was already taught in lecture. You have seen that I sped up the pace during the last three or four lectures. Although this shrunk the gap to the material assigned for study, you still are ahead if you did the reading as requested.
- Now go and reread the Advice web page of the course site. See how it fits in with what you just have read here and experienced in class and follow it more closely!

New reading assignments:

Reading assignment 1 - due Tuesday, April 11:

a. Reread carefully all of MF ch.10.1 (except for the optional chapter 10.1.6). Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for

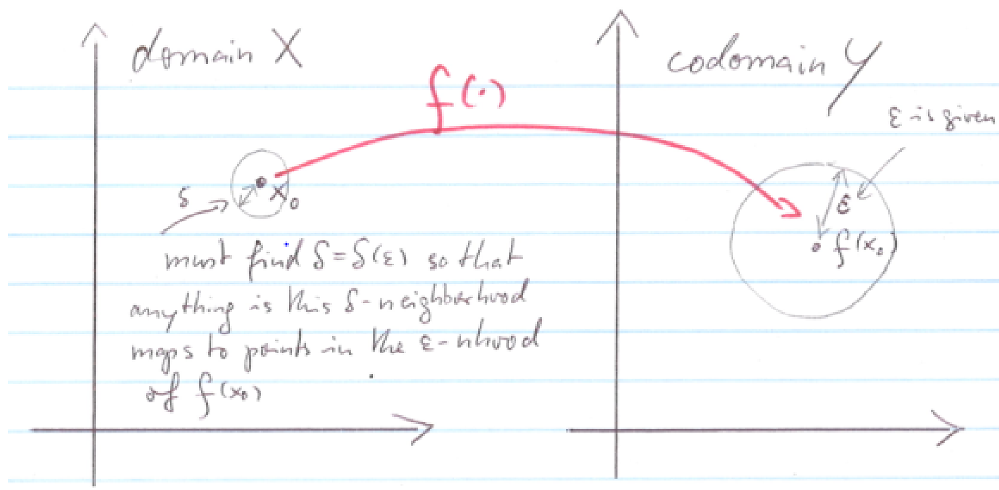
- open sets and neighborhoods (ch.10.1.3)
- convergence, expressed with nhoods (the end of def.10.9 in ch.10.1.4)
- metric subspaces (ch.10.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \uplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^\complement$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^\complement$ and A_1. What does $N_\varepsilon^A(x_j)$ look like?

- Contact sets, closed sets and closures (ch.10.1.9): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$? Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize def.10.21, thm 10.6 and thm.10.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

Reading assignment 2 - due Thursday, April 13:

a. Reread carefully MF ch.10.2.1 - 10.2.2 and reread MF ch.10.2.3 - 10.2.4. Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



Reading assignment 3 - due Saturday, April 15:

a. Read carefully the remainder of MF ch.10.2. You will need to know the essentials of linear functions between vector spaces. If you did not take a linear algebra then review MF ch.9.

Reading assignment 4 - due: Tuesday, April 18:

a. Read carefully MF ch.10.3 up to and including example 10.13 (Alternating series). How does what you see here fit in with what you learned about series in single variable calculus?

There is a written assignment on the next page.

Written assignment 1 – worth 5 points, graded only ONCE:
Prove MF prop.9.10: Let $X \neq \emptyset$. Then

$$\| \cdot \|_\infty : \mathscr{B}(X, \mathbb{R}) \to \mathbb{R}_+, \quad h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$$

is a norm on $\mathscr{B}(X, \mathbb{R})$.

Hints:

a. to prove positive definiteness and absolute homogeneity, you must show what happens for a frozen $x_0 \in X$ as a stepping stone.

b. For the triangle inequality, go on a treasure hunt in MF ch.8 and look at the material before the definition of tail sets of a sequence.