Math 330 Section 2 - Spring 2017 - Homework 18

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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 - ch.13 (ch.7 carefully until before thm.7.17, ch.11 until cor.11.23)
- B/G Appendix A: Continuity and Uniform Continuity

MF lecture notes:
- ch.1 - ch.2, ch.4 - ch.10 (skip the proof of prop.7.3 and Riemann's Rearrangement Theorem)
- MF ch.11 through thm.11.6 (Compact metric spaces are sequence compact).
- ch.16 (addenda to B/G text)

Other material:
- B/K lecture notes ch.1 – section 1, ch.4.1, ch.4.2
  (optional reading – good for examples, improved understanding)
- Stewart Calculus: "The Precise Definition of a Limit" (ch.1.7 in the 7th edition).

New reading assignments:

Reading assignment 1 - due Monday, May 1:

a. Read carefully the remainder of MF ch.11.

Reading assignment 2 - due Tuesday, May 2:

a. Go back to B/G Appendix A: Continuity and Uniform Continuity and try to find for each proposition and theorem the often more general vesion in MF ch.10 and ch.11.

Reading assignment 3 - due Wednesday, May 3:

a. Read carefully MF ch.12.1 and 12.2. Those who have not taken linear algebra will have to go back to ch.9 and reread ch.9.2.1 (Vector spaces: Definition and Examples) about subspaces, linear spans, independence, bases, ...

Reading assignment 4 - due Friday, May 5:

a. Read carefully MF ch.12.3 but SKIP lemmata 12.4 and 12.5.

Written assignment 1:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$.

Hints:

a. What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?
b. $x^2 - 1 = (x+1)(x-1)$.
c. Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $\min(\varepsilon, 1)$ and restrict your search to $\delta < 1$. What kind of bounds do you then get for $|x^2 - 1|$, $|x+1|$, $|x-1|$?
d. Put all the above together. Can you see why, for "small" δ, you obtain $|f(x) - f(x_0)| \leq 3\delta$?

Written assignment 2:

Prove MF prop.10.15 (Opposite of continuity) (ch.10.2.2: Continuity of Constants and Sums and Products)

A sequence $(x_k)_k$ with values in (X, d) does not have $L \in X$ as its limit if and only if there exists some $\varepsilon > 0$ and $n_1 < n_2 < n_3 < \cdots \in \mathbb{N}$ such that $d(x_{n_j}, L) \geq \varepsilon$ for **all** j. $\square$