Math 330 Section 3 - Fall 2017 - Homework 04

Published: Thursday, August 31, 2017 *Running total:* 18 *points*
Last submission: Friday, September 15, 2017

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1, ch.2 except the material on gcd(m, n), all of ch.3

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 (Functions and relations) of the MF document until before ch.4.2.2
ch.13.1 up to and including example 13.5
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 4: (Labor Day)

a. Continue carefully reading ch.4 of the MF doc up to and including ch.4.2.5 (Operations on Real Functions)
b. Read carefully B/G ch.4.1 and 4.2.

Reading assignment 2 - due: Wednesday, September 6:

a. Read carefully the remainder of B/G ch.4.

Reading assignment 3 - due Friday, September 8:

a. Read carefully B/G ch.5.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Prove B/G Prop. 4.6(iii) using induction: Given the definition of "Power" between props 4.5 and 4.6, prove that if $b \in \mathbb{Z}$ and $m, k \in \mathbb{Z}_{\geq 0}$ then

$$(b^m)^k = b^{mk}$$

You may use everything up to and including Prop.4.6(ii). Note that the proof of Prop.4.6(ii) provides an excellent template for your own proofs using induction.

Written assignment 2: Prove B/G Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including Prop.4.7.

Written assignment 3: Prove B/G Prop. 4.16(i) by induction on c: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \leq b < c$. Then

$$\sum_{j=a}^{c} x_j = \sum_{j=a}^{b} x_j + \sum_{j=b+1}^{c} x_j.$$

For this proof use the generalized definition of "Σ" given in MF ch.16.4.1 instead of the one given in B/G p.34, 35!

Hints: Think carefully about the base case: If $a = 5$, how would you choose b and c? If $a = 28$, how would you choose b and c? For general a, how would you choose b and c?