Math 330 Section 3 - Fall 2017 - Homework 07

Published: Thursday, September 14, 2017 *Running total:* 34 *points*
Last submission: Friday, September 29, 2017
Update Sept 19: *Requirement for written assignments 2 and 3: You are* ***not*** *allowed to use MF prop 6.5.*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1, ch.2 except the material on gcd(m, n), all of ch.3 - 5, ch.6.1.

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 6
ch.13.1 up to and including example 13.5
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 18:

- **a.** Read carefully B/G ch.9.1 (Injections and Surjections). You have encountered almost all of that material in MF ch.4.
- **b.** Read carefully B/G ch.6.2 (The Division Algorithm).
- **c.** Read the corresponding parts of MF ch.16.

Reading assignment 2 - due: Wednesday, September 20:

- **a.** Read carefully the remainder of B/G ch.6.
- **b.** Read the corresponding parts of MF ch.16.

Reading assignment 3 - due Friday, September 22:

- **a.** Read carefully B/G ch.7.1 and ch.7.2 up to and including thm.7.17, but skip the proof of that theorem. We will not talk in lecture about the remainder of B/G ch.7.
- **b.** Read the corresponding parts of MF ch.16.

Written assignment 1: (One point each for **a** and **b**) Prove formula b of De Morgan's Law:
Let there be a universal set Ω.
Then for any indexed family $(A_\alpha)_{\alpha\in I}$ of sets: $\left(\bigcap_\alpha A_\alpha\right)^{\complement} = \bigcup_\alpha A_\alpha^{\complement}$

a: Prove "$\subseteq$". **b:** Prove "$\supseteq$".

You are **not** allowed to use MF prop 6.5 (Indirect image and fibers of f) which states, among other things, that

$$x_1 \sim x_2 \Leftrightarrow f(x_1) = f(x_2)$$

defines an equivalence relation. You are encouraged to review the material in MF doc ch.4-6 and B/G ch.6.1 on equivalence relations before attacking those problems.

Written assignment 2: (One point each for **a** and **b**)

Let $L : \mathbb{R}^2 \longrightarrow [0, \infty[$ be the function which assigns to a vector $\vec{x} = (x_1, x_2) \in \mathbb{R}^2$ its "length" $L(\vec{x}) := \sqrt{x_1^2 + x_2^2}$.

For two such vectors $\vec{x} = (x_1, x_2)$ and $\vec{y} = (y_1, y_2)$ we write $\vec{x} \sim \vec{y}$ iff $L(\vec{x}) = L(\vec{y})$.

a: Prove that $\sim$ is indeed an equivalence relation for $\mathbb{R}^2$.
b: Three of the following 4 points belong to the same equivalence class: $(0, 2)$, $(1, 1)$, $(2, 0)$, $(\sqrt{2}, \sqrt{2})$ Which ones?.

Written assignment 3: (One point each for **a** and **b**)

Let X, Y be two nonempty sets and let $f : X \longrightarrow Y$. For $a, b \in X$ we write $a \sim b$ iff $f(a) = f(b)$.

a: Prove that $\sim$ is indeed an equivalence relation for X.
b: Write $[x]_f$ for the equivalence class of $x \in X$ with respect to "$\sim$". Express $[x]_f$ in terms of the function f: $[x]_f = \{x' \in X : f(x') .....??.......\}$. (I do not want to see "$[x]_f = \{x' \in X : x' \sim x\}$".)