Math 330 Section 3 - Fall 2017 - Homework 08

Published: Thursday, September 21, 2017 *Running total: 37 points*
Last submission: Friday, October 6, 2017
Update Sept 28: *Yet more hints for written assignments 1 and 2: Added equations (16.10) and (16.12).*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1, ch.2 except the material on gcd(m, n),
all of ch.3 - 6, ch.7 (skip after thm.7.17), ch.9.1

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 6
ch.13.1 up to and including example 13.5
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 25:

- **a.** Read carefully the very end of B/G ch.2: the material on gcd(m, n). This is needed for B/G ch.6.4 (Prime Numbers).
- **b.** Read carefully B/G ch.8.1–8.2. Much of it is just a repetition of the rules of arithmetic and ordering that already were discussed in ch.1 and 2 for integers, but beware of the differences (like division)!
- **c.** Read the corresponding parts of MF ch.16.

Reading assignment 2 - due: Wednesday, September 27:

- **a.** Read carefully the remainder of B/G ch.8.
- **b.** Finish B/G ch.9.
- **c.** Read the corresponding parts of MF ch.16.

Reading assignment 3 - due Friday, September 29:

- **a.** Read carefully MF ch.8.1 (Minima, Maxima, Infima and Suprema).
- **b.** Read carefully B/G ch.10.1–10.3.
- **c.** Read the corresponding parts of MF ch.16.

Written assignments:

Do not use induction for any of those assignments. It would only make your task more difficult!

#1 and #2 are about proving B/G thm.6.13 (Division algorithm for integers): Let $n \in \mathbb{N}$ and $m \in \mathbb{Z}$. There exists a unique combination of two integers q ("quotient") and r ("remainder") such that

$$m = n \cdot q + r \quad \text{and } 0 \leq r < n.$$

Written assignment 1:

Prove uniqueness of the "decomposition" $m = qn + r$: If you have a second such decomposition $m = \tilde{q}n + \tilde{r}$ then show that this implies $q = \tilde{q}$ and $r = \tilde{r}$. Start by assuming that $r \neq \tilde{r}$ which means that one of them is smaller than the other and take it from there.

Written assignment 2:

Much harder than #1: Prove the existence of q and r.

Hints for #2: Review the Well-Ordering principle from B/G ch.2. Its use will give the easiest way to prove this theorem. Remember we extended the well-ordering principle as follows in MF thm.16.1 (Generalization of the Well-Ordering Principle): Let $A \subseteq \mathbb{Z}$ have lower bounds (which is especially true if $A \subseteq \mathbb{N}$ or $A \subseteq \mathbb{Z}_{\geq 0}$). If $A \neq \emptyset$ then A has a minimum.

Apply the above to the set $A := A(m,n) := \{x \in \mathbb{Z}_{\geq 0} : x = m - kn$ for some $k \in \mathbb{Z}\}$.

Hint for both #1 and #2: MF prop.16.8 and cor.16.3 from ch.16.6.2 (The Division Algorithm) will come in handy in connection with $0 \leqq r < n$: If $m, n \in \mathbb{Z}_{\geq 0}$ then

$$|n - m| \leqq \max(m,n), \quad \text{i.e.,} \tag{16.10}$$

$$-\max(x,y) \leqq x - y \leqq \max(x,y), \tag{16.11}$$

$$-n < y - x < n. \tag{16.12}$$

Written assignment 3:

Prove prop.6.16, p.59 of B/G: Let $m \in \mathbb{Z}$. Then either m is even or $m+1$ is even.

Hint: Apply the division algorithm with $n = 2$. One of the preceding propositions in B/G ch.6 will make things really easy.