Math 330 Section 3 - Fall 2017 - Homework 09

Published: Thursday, September 28, 2017 *Running total:* 40 *points*
Last submission: Friday, October 13, 2017

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 – ch.6, ch.7 (skip after thm.7.17), ch.8 – 9, ch.10.1–10.3

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 6, ch.8.1 (Minima, Maxima, Infima and Suprema),
ch.13.1 up to and including example 13.5,
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

New reading assignments:

Reading assignment 1 - due Monday, September 25:

- **a.** Read Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- **b.** Read carefully B/G ch.10.4 (Limits).

Reading assignment 2 - due: Wednesday, September 27:

- **a.** Exam prep!

Reading assignment 3 - due Friday, September 29:

- **a.** Read carefully MF ch.8.2 (Convergence and Continuity in $\mathbb{R}$).
- **b.** Read carefully the end of B/G ch.10.

Written assignment 1:

Prove B/G Prop.7.1 using induction: If $n \in \mathbb{N}$ then $n < 10^n$. You may use the fact that 10 (defined as $9 + 1$) satisfies $0 < 1 < 2 < 10$. Justify your inequalities referring to B/G prop. 2.7(i) - 2.7(iv).

Written assignment 2 (One point each for parts a and b.)

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A \;:= A(n) \;:=\; \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) \;:=\; \min(A).$$

B/G prop.7.3 states that, for all $n \in \mathbb{N}$, $\nu(n) = k \iff 10^{k-1} \leqq n < 10^k$.

2a. Prove "$\Rightarrow$" of B/G prop.7.3.

2b. Prove "$\Leftarrow$" of B/G prop.7.3.

The math for assignment 2 is easy but you may find it hard to write down a proof that meets my demands for precision.

Hints for #2 and #3: 1) I gave the set a name (A) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** A: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)