Math 330 Section 3 - Fall 2017 - Homework 11

Published: Thursday, October 12, 2017 *Running total:* 45 *points*
Last submission: Friday, October 27, 2017

***Note**: This is the Fall break assignment (no lecture on Mon, 10/16).*

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 – ch.6, ch.7 (skip after thm.7.17), ch.8 – 10

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 6, ch.8.1 and 8.2
ch.8.1 – 8.4, except: Skip the proofs of prop.8.13, 8.14, 8.15, cor.8.2, thm.8.2; skip rem.8.6
ch.13.1 up to and including example 13.5,
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

Reading assignment 1 - due Monday, October 16: (Fall break)

- **a.** Read carefully MF ch.7.1 until before prop.7.1 (1 1/2 pages) That's just enough so you can understand rem.8.6 which is part of the next assignment ...
- **b.** Read carefully the end of MF ch.8 (that's ch.8.5).
- **c.** Read carefully B/G ch.11.1 (Rational Numbers).

Reading assignment 2 - due: Wednesday, October 18:

- **a.** Read carefully B/G ch.11.2 (Irrational Numbers).
- **b.** Read B/G ch.11.3 (Quadratic Equations). Skip all proofs!

Reading assignment 3 - due Friday, October 20:

- **a.** Read carefully B/G ch.12 (Decimal Expansions) through prop.12.7.

Written assignment 1:
Prove MF doc excercise 8.7: Let $x_n := (-1)^n$ for $n \in \mathbb{N}$. Prove that $\liminf_n x_n = -1$ and $\limsup_n x_n = 1$ by working with the tailsets of that sequence. Do not use anything after definition 8.14!
Hint: What is α_n and β_n?

Written assignment 2:
Prove B/G prop.10.10(iv): $x, y \in \mathbb{R} \Rightarrow |x - y| \geq \big||x| - |y|\big|$.

Hint #1: To show this use the following proposition (very similar to B/G prop.10.8(v)).

Proposition. (B/G prop.10.8(v)) Let $a, b \in \mathbb{R}$ such that both **#1)** $-a \leqq b$ and **#2)** $a \leqq b$. Then $|a| \leqq b$.

Proof of proposition:
Case 1) $a \geqq 0$: It follows from **#2** that $|a| = a \leqq b$ which is what we had to show.
Case 2) $a < 0$: It follows from **#1** that $|a| = -a \leqq b$ which is what we had to show. ■.

Hint #2: first use the triangle inequality on $|x| = |(x - y) + y|$ and then on $|y| = |(y - x) + x|$. See what you get for $a := |x| - |y|$ and $b := |x - y|$.