Math 330 Section 3 - Fall 2017 - Homework 14

Published: Thursday, November 2, 2017 *Running total:* 53 *points*
Last submission: Friday, November 17, 2017

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 – ch.6, ch.7 (skip after thm.7.17), ch.8 – 13.

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 7,
ch.8, except: Skip the proofs of prop.8.13, 8.14, 8.15, cor.8.2, thm.8.2;
ch.9 except optional ch.9.2.3, ch.10 through 10.1.3,
ch.13.1 up to and including example 13.5,
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:
- Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- Paul Dawkins: Linear Algebra (lecture notes): Vector Spaces – Subspaces p.193: def.1, thm 1 – Span, p.202: def 1, def 2, thm 1 – Linear independence, p.210: def.1 – Basis & dimension, p.220: def.1, thm 2, def 2, thm 3.

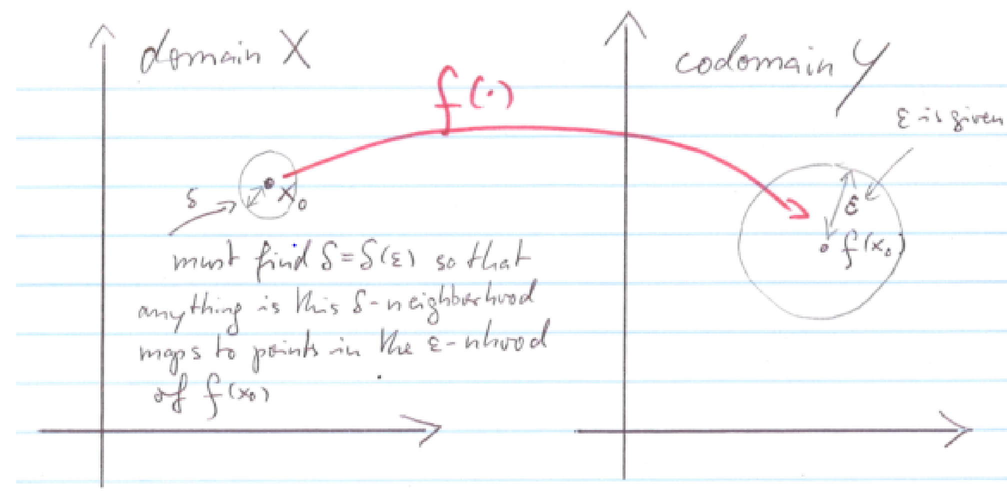
Supplementary instructions for reading MF ch.10.1, and ch.10.2:

a. MF ch.10.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for
- open sets and neighborhoods (ch.10.1.3)
- convergence, expressed with nhoods (the end of def.10.11 in ch.10.1.4)
- metric and topological subspaces (ch.10.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What does $N_\varepsilon^A(x_j)$ look like?

- Contact points, closed sets and closures (ch.10.1.9): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 10.6 and thm.10.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.10.2: Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



New reading assignments:

Reading assignment 1 - due Monday, November 6:

a. Read carefully the remainder of MF ch.10.1 (The Topology of Metric Spaces).

- Optional ch.10.1.6 (Neighborhood Bases): Skip the proofs. None of this will be on any exam or quiz but I will refer to the material when needed in later chapters.

Reading assignment 2 - due: Wednesday, November 8:

a. Read carefully MF ch.10.2 (Continuity).

Reading assignment 3 - due Friday, November 10:

a. Read carefully MF ch.10.3 (Function Sequences and Infinite Series). Skip the proofs of lemma 10.2 and thm.10.18 (Riemann's Rearrangement Theorem) but be able to write down that theorem and **be sure to understand the corollaries that follow** and their proofs!

Written assignments on next page!

Written assignment 1 (worth 3 points, one each for **a**, **b**, **c**).

Prove MF prop.9.10: Let $X \neq \emptyset$. Then

$$\|\cdot\|_\infty : \mathscr{B}(X,\mathbb{R}) \to \mathbb{R}_+, \quad h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$$

is a norm on $\mathscr{B}(X,\mathbb{R})$ in the sense of def.9.14, i.e., it satisfies **a**. positive definiteness, **b**. absolute homogeneity, **c**. the triangle inequality (for norms).

Hints: for problem 2:

- **a.** to prove positive definiteness and absolute homogeneity, you must show what happens for a frozen $x_0 \in X$ as a stepping stone.
- **b.** For the triangle inequality, go on a treasure hunt in MF ch.8.1.

Written assignment 2 (one point only) Do MF exercise 9.4:
Prove that the p–norm (see def.9.15) is a norm on $\mathbb{R}^n$ for the special case $p = 1$:

$$\|\vec{x}\|_1 = \sum_{j=1}^{n} |x_j| \quad \square$$