Math 330 Section 3 - Fall 2017 - Homework 17

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Update Dec 4, 2017

Every student will get a point for assignment 2 because I had messed it up, talking about continuity rather than convergence!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 – ch.6, ch.7 (skip after thm.7.17), ch.8 – 13.
B/G Appendix A (Continuity and Uniform Continuity)

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets), ch.4 – 7,
ch.8, except: Skip the proofs of prop.8.13, 8.14, 8.15, cor.8.2, thm.8.2;
ch.9 except optional ch.9.2.3, ch.10 except optional ch.10.1.6, ch.11.1 – 11.4,
ch.13.1 up to and including example 13.5,
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:

- Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- Paul Dawkins: Linear Algebra (lecture notes): Vector Spaces – Subspaces p.193: def.1, thm 1 – Span, p.202: def 1, def 2, thm 1 – Linear independence, p.210: def.1 – Basis & dimension, p.220: def.1, thm 2, def 2, thm 3.

New reading assignments:

Reading assignment 1 - due Monday, November 27:

a. Read carefully MF ch.11.5 and 11.6 (the remainder of ch.11).

Reading assignment 2 - due: Wednesday, November 29:

a. Read carefully MF ch.12.1 – 12.3.

Reading assignment 3 - due Friday, December 1:

a. Read carefully MF ch.12.4 – 12.5.1. Read ch.12.5.1 even though it is optional!

Written assignment 1:

Let $f(x) = x^2$. Prove by use of "ε-δ continuity" that f is continous at $x_0 = 1$.

Hints:

a. What does $d(x, x_0) < \delta$ and $d(f(x), f(x_0) < \varepsilon$ translate to?

b. $x^2 - 1 \ = \ (x+1)(x-1)$.

c. Only small neighborhoods matter: Given $\varepsilon > 0$ try to find δ that works for $0 < \varepsilon < 1$. Restrict your search to $\delta < 1$. What kind of bounds do you get for $|\, x^2 - 1|$, $|x+1|$, $|x-1|$? if $0 < \delta < 1$?

d. Put all the above together. Show that you obtain $|f(x) - f(x_0)| \leq 3\delta$?. How then do you choose δ when you consider $\varepsilon as given$?

Written assignment 2:

MF Exercise 10.23: Prove MF prop.10.7 (Opposite of convergence):

A sequence $(x_k)_k$ with values in (X, d) does not have $L \in X$ as its limit if and only if there exists some $\varepsilon > 0$ and $n_1 < n_2 < n_3 < \cdots \in \mathbb{N}$ such that $d(x_{n_j}, L) \geq \varepsilon$ for **all** j.