Math 330 Section 3 - Fall 2017 - Homework 18

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Update Dec 7, 2017

Provided Hwk 18 Lemma as an aid to solve assignment 1a.

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
all of ch.1 – ch.6, ch.7 (skip after thm.7.17), ch.8 – 13.
B/G Appendix A (Continuity and Uniform Continuity)

MF lecture notes:
ch.1; ch.2 except optional ch.2.2.1 (Rings & Algebras of Sets),
ch.4 – 7,
ch.8, except: Skip the proofs of prop.8.13, 8.14, 8.15, cor.8.2, thm.8.2;
ch.9 except optional ch.9.2.3, ch.10 except optional ch.10.1.6, ch.11, ch.12 through ch.12.5.1
ch.13.1 up to and including example 13.5,
ch.16 (Addenda to B/G): the chapters corresponding to what has been assigned from B/G.

B/K lecture notes:
ch.1.1 (Introduction to sets) (optional)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions (optional)

Other:

- Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.
- Paul Dawkins: Linear Algebra (lecture notes): Vector Spaces – Subspaces p.193: def.1, thm 1 – Span, p.202: def 1, def 2, thm 1 – Linear independence, p.210: def.1 – Basis & dimension, p.220: def.1, thm 2, def 2, thm 3.

No new reading assignments, BUT review MF ch.11 on compactness and ch.12 until ch.12.5.1 on applications of Zorn's Lemma.

Written assignments: Next page!

Written assignment 1:
One point each for 1a and 1b

Let $N \in \mathbb{N}$. Let $X := \{x_1, x_2, \ldots, x_N\}$ be a finite set with a metric $d(\cdot,\cdot)$ (so (X, d) is a metric space).

- **a.** Prove that X is sequence compact.
- **b.** Let (Y, d') be a second metric space and $f : X \to Y$. Prove that $\left(f(X), d'\big|_{f(X)\times f(X)}\right)$ is compact.

Hint to a: ANY sequence in X possesses a constant subsequence. Prove this indirectly. You may use Hwk 18 Lemma below but **you must reference it** under that name.
Hint to b: Apply part **a.**

Hwk 18 Lemma: Let $(z_n)_n$ be a sequence in X. Let $a \in X$ and let $I \subseteq \mathbb{N}$ be an **infinite** set of indices such that $z_i = a$ for all $i \in I$.

Then there exists a subsequence $(z_{n_j})_j$ of $(z_n)_n$ such that $n_j \in I$ for all $j \in \mathbb{N}$ (i.e. there is $n_1 < n_2 < \ldots$ such that $n_j \in I$ and hence $z_{n_j} = a$ for all j).