Math 330 Section 5 - Spring 2018 - Homework 03

Published: Thursday, January 25, 2018 *Running total:* 17 *points*
Last submission: Wednesday, January 31, 2018 ***NO RESUBMISSIONS***
*(that is two days **before** the last submission date for hwk 2!)*

This homework is published concurrently with homework 4

Helpful hints:

a. No matter what A stands for, it is never true that $A = \{A\}$. Not even if $A = \emptyset$ (the empty set): $\{A\} = \{\emptyset\}$ is a set which contains an element (exactly one): The empty set! Because $\{\emptyset\}$ is not empty it follows that $\emptyset \neq \emptyset$. By the way: It is true that $\emptyset \subseteq \{\emptyset\}$!

b. **Correct**: No matter what A stands for, it is never true that $A \in A$. Again, not even if $A = \emptyset$ (the empty set): The empty set contains nothing at all; in particular, it does not contain any set; in particular, it does not contain the set that has no elements, i.e., the empty set.

c. **CAREFUL HERE**: It is possible to have both $a \in U$ and $\{a\} \in U$. Matter of fact, the first assignment of this homework contains such an example.

Written assignments 1-5 Partial credit will be given. You can earn as many as 10 points!

Note the following:

A. In the MF doc refer to example 5.4 for the preliminary definition of the size of a set S:
If S is finite then $|S|$ is the number of elements of S, otherwise $|S| = \infty$.

B. Refer to MF doc def.2.17 (Preliminary definition: cartesian product) for the definition of $X \times Y$.

Written assignment 1:

Let $S = \{3, 5, \{3, 5\}, \{5\}\}$. True or false?

a. $\{5\} \subseteq S$ **c.** $\{\{5\}\} \subseteq S$ **e.** $\{3\} \subseteq S$ **g.** $3 \subseteq S$
b. $\{5\} \in S$ **d.** $\{\{5\}\} \in S$ **f.** $\{3\} \in S$ **h.** $3 \in S$

Written assignment 2:

Find the cardinality of each of the following sets:

a. $A = \{x, \{x\}, y, \{x\}, \{x, y\}\}$ **c.** $C = \{j, k, j, k, j\}$ **e.** $E = \{e^x : x \in \mathbb{R}\}$
b. $B = \{a, \{a\}, \{b\}\}$ **d.** $D = \{4q^2 : q \in \mathbb{Z}\}$ **f.** $F = \{(-1)^m : m \in \mathbb{Z}\}$

Written assignment 3:

Let $X = \{x, y, \{x\}, \{x, y\}\}$ and $Y = \{x, \{y\}\}$. True or false?

a. $x \in X \cap Y$ **c.** $x \in X \cup Y$ **e.** $x \in X \setminus Y$ **g.** $x \in X \Delta Y$
b. $\{y\} \in X \cap Y$ **d.** $\{y\} \in X \cup Y$ **f.** $\{y\} \in X \setminus Y$ **h.** $\{y\} \in X \Delta Y$

Written assignment 4:

Let $X = \{x, y\}$ and let $Y = \{1, 2, 3\}$.

a. What is $X \times Y$?
b. What is $Y \times X$?
c. What is $\text{card}(X \times Y)$?
d. What is $\text{card}(Y \times X)$?
e. Is $(x, 3) \in X \times Y$?
f. Is $(x, 3) \in Y \times X$?
g. Is $3 \cdot x \in X \times Y$?
h. Is $2 \cdot y \in Y \times X$?

Written assignment 5:

Let $X = \{8\}$.

a. What is 2^X?
b. What is $2^{\left(2^X\right)}$?