Math 330 Section 5 - Spring 2018 - Homework 04

Published: Thursday, January 11, 2018 *Running total:* 20 *points*
Last submission: Friday, February 9, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1, ch.2 except the material on gcd(m, n), all of ch.3

MF lecture notes:
- ch.1 – ch.3
- ch.17 (Addenda to B/G): the chapters corresponding to what has been assigned so far from B/G.

B/K lecture notes:
- ch.1.1 (Introduction to sets) (optional)
- ch.1.1 (Introduction to sets) (optional) ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments:

Reading assignment 1 - due Monday, January 29:

a. Carefully read B/G ch.4.1 – 4.4.

Reading assignment 2 - due: Wednesday, January 31:

a. Carefully read the remainder of B/G ch.4.
b. Review MF ch.2.3 (functions and cartesian products). Most of it is known from MF ch.2.
c. Carefully read B/G ch.5. Most of it is known from MF ch.2.

Reading assignment 3 - due Friday, September 8:

a. Carefully read MF ch.5.1 – 5.3.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1: Prove B/G Prop. 4.6(iii) using induction: Given the definition of "Power" between props 4.5 and 4.6, prove that if $b \in \mathbb{Z}$ and $m, k \in \mathbb{Z}_{\geq 0}$ then

$$(b^m)^k = b^{mk}$$

You may use everything up to and including Prop.4.6(ii). Note that the proof of Prop.4.6(ii) provides an excellent template for your own proofs using induction.

Written assignment 2: Prove B/G Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including Prop.4.7.

Written assignment 3: Prove B/G Prop. 4.16(i) by induction on c: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \leq b < c$. Then

$$\sum_{j=a}^{c} x_j \;=\; \sum_{j=a}^{b} x_j \;+\; \sum_{j=b+1}^{c} x_j.$$

For this proof use the generalized definition of "Σ" given in MF ch.17.4.1 instead of the one given in B/G p.34, 35!

Hints: Think carefully about the base case: If $a = 5$, how would you choose b and c? If $a = 28$, how would you choose b and c? For general a, how would you choose b and c?