# Math 330 Section 5 - Spring 2018 - Homework 06 

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Running total: 29 points
Last submission: Friday, February 14, 2018 NO RESUBMISSIONS

New reading assignments: None: They came with homework 5.

The written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

## Written assignment 1:

Injectivity and Surjectivity

- Let $f: \mathbb{R} \longrightarrow\left[0, \infty\left[; \quad x \mapsto x^{2}\right.\right.$.
- Let $g:\left[0, \infty\left[\longrightarrow\left[0, \infty\left[; \quad x \mapsto x^{2}\right.\right.\right.\right.$.

In other words, $g$ is same function as $f$ as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with true or false.
a. $f$ is surjective
b. $\quad f$ is injective
c. $g$ is surjective
d. $g$ is injective

If your answer is false then give a specific counterexample.

## Written assignment 2:

Find $f: X \longrightarrow Y$ and $A \subseteq X$ such that $f\left(A^{\complement}\right) \neq f(A)^{\complement}$. Hint: use $f(x)=x^{2}$ and choose $Y$ as a one element only set (which does not leave you a whole lot of choices for $X$ ). See MF example 5.17 on approx. p. 90 .

## Written assignment 3:

Let $f:]-10.10\left[\longrightarrow \mathbb{R} ; \quad x \mapsto x^{2}\right.$.
a. what is the range of $f$ ?
b. Is $f$ injective?
c. Is $f$ surjective?
d. $f(\{1\} \cup[4,6]=$ ?
e. $f([2,5]) \cap f([4,7])=$ ?
f. $f^{-1}([4,25]) \cap f^{-1}([16,49])=$ ?

## Written assignment 4:

You will learn later in this course that
injective $\circ$ injective $=$ injective,
surjective $\circ$ surjective $=$ surjective.
The following illustrates that the reverse is not necessarily true.
Find functions $f:\{a\} \longrightarrow\left\{b_{1}, b_{2}\right\}$ and $g:\left\{b_{1}, b_{2}\right\} \longrightarrow\{a\}$ such that $h:=g \circ f:\{a\}$ is bijective but such that it is not true that both $f, g$ are injective and it is also not true that both $f, g$ are surjective.

Hint: There are not a whole lot of possibilities. Draw possible candidates for $f$ and $g$ in arrow notation as on p.118. You should easily be able to figure out some examples. Again, think simple and look at MF example 5.17 on approx. p. 90 .

