Math 330 Section 5 - Spring 2018 - Homework 08

Published: Thursday, February 15, 2018 *Running total:* 37 *points*
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Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

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B/G (Beck/Geoghegan) Textbook:

- all of ch.1, ch.2 except the material on gcd(m, n), all of ch.3 – ch.7 (ch.7: skip all after thm.7.17), ch.8, ch.9.1

MF lecture notes:

- ch.1 – ch.3, ch.5, ch.6 (skip ch.6.3), ch.7
- ch.17 (Addenda to B/G): the chapters corresponding to what has been assigned so far from B/G.

B/K lecture notes (optional):

- ch.1.1 (Introduction to sets)
- ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments:

Reading assignment 1 - due Monday, February 19:

a. Read Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index. If you used a different calculus book, bring it and see me.

b. Read carefully MF ch.9.1

Reading assignment 2 - due: Wednesday, February 21:

a. Read carefully MF ch.9.2.

Reading assignment 3 - due Friday, February 23:

a. Read carefully MF ch.9.3.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1:

Define $\nu : \mathbb{Z}_{\geqq 0} \longrightarrow \mathbb{Z}_{\geqq 0}$ as follows: $\nu(0) := 0$. For $n \in \mathbb{N}$ proceed as follows: Let

$$A \ := A(n) \ := \ \{t \in \mathbb{N} : n < 10^t\}; \qquad \text{define } \nu(n) \ := \ \min(A).$$

B/G prop.7.3 states that, for all $n \in \mathbb{N}$, $\nu(n) = k \ \Leftrightarrow \ 10^{k-1} \leqq n < 10^k$. Prove "$\Leftarrow$".

Hints for #1:
1) I gave the set a name (A) on purpose: this allows you to express with minimal effort fragments such as "$x \in A$", "$x \notin A$", "because $\nu(m) = \min(A)$", ...
2) You may use without proof the **"no gaps property" of** A but you must refer to it: if $x, y \in \mathbb{N}$ and $x \in A$ and $y > x$ then $y \in A$. (would you be able to figure out why?)

Written assignment 2:

Let $x, y \in \mathbb{R}$ such that $x < y$. Let $z := (x+y)/2$. Prove that $x < z < y$. **Hint**: Prove first that $2x < x + y < 2y$. Then use B/G prop.8.37(ii): $\big[\alpha > 0$ and $\alpha u < \alpha v \ \Rightarrow \ u < v\big]$ to show that $x < z < y$.

Written assignment 3:

Similar to B/G prop.8.49: Let $A \subseteq \mathbb{R}$ be a nonempty subset of $\mathbb{R}$ which is bounded below. Prove that if $\inf(A)$ exists and if $\inf(A) \in A$ then $\min(A)$ exists and $\min(A) = \inf(A)$.