Math 330 Section 5 - Spring 2018 - Homework 10

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Update March 13, 2018
Rewrote #2 so that you only need to know about convergence.

Update March 15, 2018
Typo in #2: "Let $x_n := (-1)^n(n + \frac{1}{n})$*" should read "Let* $x_n := (-1)^n(1 + \frac{1}{n})$*" (as was done in lecture).*

Update March 20, 2018
Improved hints for #2

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1, ch.2 except the material on gcd(m, n), all of ch.3 – ch.7 (ch.7: skip all after thm.7.17), ch.8, ch.9.1, 9.2

MF lecture notes:
- ch.1 – ch.3, ch.5, ch.6 (skip ch.6.3), ch.7, ch.9.1 - 9.3
- ch.17 (Addenda to B/G): the chapters corresponding to what has been assigned so far from B/G.

B/K lecture notes (optional):
- ch.1.1 (Introduction to sets)
- ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, March 12:

a. Read carefully B/G ch.10.3. and ch.10.4 Ch.10.4 on limits is very long, but there is a lot of overlap with MF ch.9.2. Be sure to see the connections between those chapters.

Reading assignment 2 - due Wednesday, March 18:

a. Read carefully MF ch.9.4. Skip the proofs of prop.9.16 – 9.18. Thm.9.2: Only read the second proof (the one that does not make use of prop.9.16 – 9.18).

Reading assignment 3 - due Friday, March 20:

a. Read carefully the end of B/G ch.10.
b. Read carefully MF ch.9.5. This chapter is brief, but very abstract!
c. MF ch.9.6 will be put on hold until we have talked about the uncountability of $\mathbb{R}$.

Written assignment 1:
Prove B/G prop.10.10(iv): $x, y \in \mathbb{R} \Rightarrow |x - y| \geq \big||x| - |y|\big|$.

Hint #1: To show this use the following proposition (very similar to B/G prop.10.8(v)).

Proposition. (B/G prop.10.8(v)) Let $a, b \in \mathbb{R}$ such that both **#1)** $-a \leqq b$ and **#2)** $a \leqq b$. Then $|a| \leqq b$.

Proof of the above proposition:
Case 1) $a \geqq 0$: It follows from **#2** that $|a| = a \leqq b$ which is what we had to show.
Case 2) $a < 0$: It follows from **#1** that $|a| = -a \leqq b$ which is what we had to show. ■.

Hint #2: first use the triangle inequality on $|x| = |(x - y) + y|$ and then on $|y| = |(y - x) + x|$. See what you get for $a := |x| - |y|$ and $b := |x - y|$.

Written assignment 2:
Let $x_n := (-1)^n(1 + \frac{1}{n})$ for $n \in \mathbb{N}$. Let $T_n := \{x_j : j \geqq n\}$ be the "tail sets" of that sequence. (see def.9.14) Let $\alpha_n := \inf(T_n)$ and $\beta_n := \sup(T_n)$ Prove that $\lim\limits_{n\to\infty} \alpha_n = -1$ and $\lim\limits_{n\to\infty} \beta_n = 1$. You may use anything from MF ch.9.1 and 9.2.

Hint: Compute α_n and β_n for $n = 1, 2, 3, 4, 5, 6$ to see the pattern: Look separately at odd and even indices to prove that both $|\beta_n - 1|$ and $|\alpha_n + 1|$ are either $\frac{1}{n}$ or $\frac{1}{n+1}$. What follows for the convergence behavior of α_n and β_n?