Math 330 Section 5 - Spring 2018 - Homework 13

Published: Saturday, April 7, 2018 *Running total:* 47 *points*
Last submission: Friday, April 20, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 – ch.7 (ch.7: skip all after thm.7.17),
- ch.8 – 13

MF lecture notes:
- ch.1 – ch.3, ch.5 – ch.8 (skip ch.6.3),
- ch.9.1 - 9.5 (see hwk 10 for exceptions to ch.9.4), ch.10.1, ch.10.2.1
- ch.17 (Addenda to B/G): the chapters corresponding to what has been assigned so far from B/G.

B/K lecture notes (optional):
- ch.1.1 (Introduction to sets)
- ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 9:

a. Finish MF ch.9: Carefully read ch.9.6
b. Read MF ch.10.2.2 (Normed Vector Spaces). This finishes ch.10 because ch.10.2.3 is optional.

Reading assignment 2 - due: Wednesday, April 11:

a. Prepare for exam.2. All of B/G ch.13 and MF ch.8.1 until before ch.8.1.1 is in scope, but ch.8.1.1 (Cardinality as a Partial Ordering) is not.

Reading assignment 3 - due Friday, April 13:

a. Carefully read MF ch.11.1.1 – 11.1.2. It is to a large degree a collection of examples of distance functions, but you will be lost if you do not study this, preferably drawing lots of pictures.

Written assignment 1:
// Use anything up-to and including MF thm.7.2 and anything in B/G ch.13 to prove MF cor.7.3:

Let the set X not be countable and let $A \subseteq X$ be countable. Then its complement $A^{\complement}$ is not countable.

Written assignment 2:
Let X be a set which contains at least 2 elements. Prove that $X^{\mathbb{N}} = \{(x_n)_{n\in\mathbb{N}} : x_j \in X \ \forall j \in \mathbb{N}\}$ (the set of all sequences with values in X) is uncountable. Do this by emulating the proof of B/G thm.13.22 or MF thm.8.5 (The real numbers are uncountable). Do not use other results about uncountable sets!

Hints:

a. Prove this indirectly by assuming that the elements of $X^{\mathbb{N}}$ can be written as a sequence $(\vec{x}_i)_{i\in\mathbb{N}}$, and construct an element $\vec{x} \in X^{\mathbb{N}}$ which is different from each $\vec{x}_i$.

b. A big obstacle to solving this problem is using the wrong notation. If you stick to the following, this should make life easier: Do as above and use an arrow superscript as I did above to distinguish $\vec{x} \in X^{\mathbb{N}}$ from $x \in X$. You need TWO indices to denote the j–th member x^i_j of the i–th member $\vec{x}_i$ of the sequence $(\vec{x}_i)_{i\in\mathbb{N}}$. If your handwriting is very neat, you can also write $x_{i,j}$ or x_{ij} instead of x^i_j, but be sure to make it easy for me to distinguish symbols which are subscripts from those that are not!