Math 330 Section 5 - Spring 2018 - Homework 14

Published: Thursday, April 12, 2018 *Running total:* 51 *points*
Last submission: Friday, April 27, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
- all of ch.1 – ch.7 (ch.7: skip all after thm.7.17),
- ch.8 – 13

MF lecture notes:
- ch.1 – ch.3, ch.5 – ch.8 (skip ch.6.3),
- ch.9 (see hwk 10 for exceptions to ch.9.4), ch.10, ch.11.1.1 – 11.1.2
- ch.17 (Addenda to B/G): the chapters corresponding to what has been assigned so far from B/G.

B/K lecture notes (optional):
- ch.1.1 (Introduction to sets)
- ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

Other:
Stewart Calculus 7ed - ch.1.7: "The Precise Definition of a Limit". If you have a newer or older edition then you may have to search through the table of contents and/or consult the index.

New reading assignments:

Reading assignment 1 - due Monday, April 16:

Carefully read MF ch.11.1.3 – 11.1.9.

Reading assignment 2 - due: Wednesday, April 18:

a. Carefully read MF ch.11.1.10
b. Carefully read MF ch.11.2.1 – 11.2.3

Reading assignment 3 - due Friday, April 20:

a. Carefully read MF ch.11.2.4 – 11.2.5
b. Carefully read MF ch.11.3.1

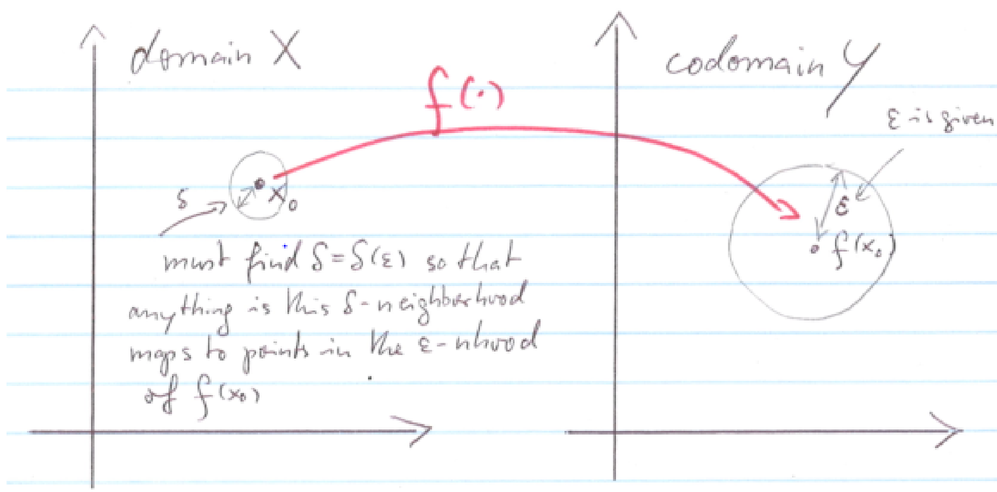
Supplementary instructions for reading MF ch.11.1, and ch.11.2:

a. MF ch.11.1: Draw as many pictures as possible to get a feeling for the abstract concepts. Use the metric spaces $(\mathbb{R}^2, d\big|_{\|\cdot\|_2})$ and $(\mathscr{B}(X,\mathbb{R}), d\big|_{\|\cdot\|_\infty})$ for this. Do these drawings in particular for
- open sets and neighborhoods (ch.11.1.3)
- convergence, expressed with nhoods (the end of def.11.11 in ch.11.1.4)

- metric and topological subspaces (ch.11.1.7): draw an irregular shaped subset $A \subseteq \mathbb{R}^2$ in two pieces $A = A_1 \biguplus A_2$ which do not overlap. Draw some points $x_j \in A$ with ε–nhoods (circles with radius ε about x_j) so that some circles are entirely in A, one with $x_j \in A_1$ which reaches into $A^{\complement}$ but not into A_2, and one with $x_j \in A_2$ which reaches both into $A^{\complement}$ and A_1. What does $N_\varepsilon^A(x_j)$ look like?
- Contact points, closed sets and closures (ch.11.1.9): Draw subsets $B \subseteq \mathbb{R}^2$ with parts of their boundary (periphery) drawn solid to indicate that points there belong to B and other parts drawn dashed to indicate that those boundary points belong to the complement. What is $\bar{B}$?
  Draw points "completetely inside" B, others "completetely outside" B, and others on the solid and dashed parts of the boundary. Which ones can you approximate from within B by sequences? Which ones can you surround by circles that entirely stay within B, i.e., which ones are interior points of B? Which ones can you surround by circles that entirely stay outside the closure of B, i.e., which ones are entirely within $\bar{B}^{\complement}$? Use those pictures to visualize the definitions in this chapter and thm 11.6 and thm.11.7.
- Now repeat that exercise with an additional set A which is meant to be a metric subspace of $\mathbb{R}^2$.

b. MF ch.11.2: Draw as many pictures as possible to get a feeling for continuity, especially if you did not take multivariable calculus and are not used to dealing with continuous/differentiable functions of more than one variable. Here is a picture.

Figure 1: ε-δ continuity



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Written assignments on next page!

Written assignment 1 (worth 3 points, one each for **a**, **b**, **c**).

Prove MF prop.10.13: Let $X \neq \emptyset$. Then

$$\| \cdot \|_\infty : \mathscr{B}(X, \mathbb{R}) \to \mathbb{R}_+, \quad h \mapsto \|h\|_\infty = \sup\{|h(x)| : x \in X\}$$

is a norm on $\mathscr{B}(X, \mathbb{R})$ in the sense of def.10.15, i.e., it satisfies **a**. positive definiteness, **b**. absolute homogeneity, **c**. the triangle inequality (for norms).

Written assignment 2 (one point only) Do MF exercise 10.7:
Prove that the p–norm (see def.10.16) is a norm on $\mathbb{R}^n$ for the special case $p = 1$:

$$\|\vec{x}\|_1 = \sum_{j=1}^{n} |x_j| \quad \square$$

Hints: for problem 2:

- **a.** to prove positive definiteness and absolute homogeneity, you must show what happens for a frozen $x_0 \in X$ as a stepping stone.
- **b.** For the triangle inequality, go on a treasure hunt in MF ch.9.1.