Math 330 Section 2 - Fall 2018 - Homework 01

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Update Aug 24, 2018

Error in assignment # 5 (was the trivial $a(bc) = a(bc)$.

Status - Reading Assignments:

In this spot you would find the previously assigned reading assignments. There are none because this is your first homework.

New reading assignments:

In the following B/G refers to the (yellow) textbook and MF refers to the Instructor's lecture notes (see the Course Material page of the course website).

Reading assignment 1 - due Wednesday, August 22(!):

a. Familiarize yourself with the entire course website!

b. Read ch.2.1 through ch.2.3 of the MF doc. You should be familiar with most if not all the material of ch.2.1 and ch.2.2. You need to familiar with the differences between natural numbers, integers, and rational numbers and have a basic understanding of why an expression like $a + b$ can be thought of as a function of two variables a and b.
Note that the material of ch.2.1 – 2.2 is considered general knowledge for anyone who has studied at least one semester of calculus, a prerequisite for this course. I will skip most of those chapters in class.

b. Read **carefully** ch.3.1 (Semigroups and Groups) of the MF doc. This is a shock treatment, and it is deliberate, as **the abstractness of the material is indicative of what you have to expect during most of the semester**. The chapter is long, but a lot of it is taken by straight line geometry in the plane. You will see a lot of definitions, and you must remember most of them, because they might come up in quizzes and exams. You will definitely have to be able to produce the definition of groups and subgroups from memory, not literally, but correctly. You will not be quizzed on the terms "semigroup" and "monoid" (but you need to remember "associativity" and "neutral element"). Try to understand example 3.5. To do so you must have read MF ch.2.3: A First Look at Functions and Sequences.

If you did not see this assignment in time for Wednesday's lecture then be sure to complete the first reading assignment by Friday, August 24.

Reading assignment 2 - due Friday, August 24:

a. Read the preface and the notes for both student and instructor in the B/G (Beck Geoghegan) text.

b. Read carefully B/G ch.1 (Integers). Relate it to MF ch.3.1 with help of MF example 3.4 a and c. The connection will become much clearer when you have studied the material of MF ch.3.2 and 3.3.

c. Read ch.1.1 (Before You Start) and ch.1.2 (How to Properly Write a Proof) of the MF document.

d. Look at the sample homework assignment which is posted on the Homework page of the course website.

e. **Optional**, but needed if you want to get an early start on the written assignments: Read the following parts of MF ch.3.2 in this order:

1. Skim def.3.6 (commutative rings with unit) and def.3.8 (Zero Divisors and Cancellation Rule), just so you understand the important one of integral domains. Skim their definition (def.3.8) too and skim MF prop.3.9.

2. Read the blue box in rem.3.4. It states the properties of an integral domain in such a fashion that they match axioms 1.1 – 1.5 in B/G ch.1. (MF prop.3.9 asserts that the cancellation rule of B/G is equivalent to the no zero divisors rule of MF rem.3.4.j.). Use references like, e.g., "rem.3.4.c", in your homework if you want to indicate that you used commutativity of $\odot$ in "that" step of your proof.

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book **up to but NOT including** the specific item you are asked to prove.

For all written assignments here assume that $R = (R, \oplus, \cdot)$ is an integral domain.

Be sure to use $\oplus$ rather than $+$ and $\odot$ rather than $\cdot$, and $\ominus$ rather than $-$. However you may write xy for $x \odot y$ (see notations 3.1.**a**). Problem 5 of this homework is an example for this.

Written assignment 1:
Prove MF Prop.3.13: Let $a \in R$. Then $(\ominus a) \oplus a = 0$.

Written assignment 2:
Prove MF Prop.3.15: Let $a, b_1, b_2 \in R$. If $(a \oplus b_1) = 0$ and $a \oplus b_2 = 0$ then $b_1 = b_2$.

Hint: You may use MF prop.3.11 – 3.14.

Hints for assignments #3 and #4:
a. Do **NOT** use commutativity: the variables appear in the same left–to–right order on both sides!
b. Obviously you'll have to utilize associativity of $\oplus$ to prove #3 and #4. Tell me me what you plug in for a, b, c in rem.3.4.**b**.

Written assignment 3:
Prove the first equation of MF Prop.3.16.**b**: Let $a, b, c, d \in R$. Then $a \oplus \big(b \oplus (c \oplus d)\big) = (a \oplus b) \oplus (c \oplus d)$.

Written assignment 4:
Let $a, b, c, d \in R$. Prove that $(a \oplus b) \oplus (c \oplus d) = \big((a \oplus b) \oplus c\big) \oplus d$. Note that this is **not** the second equation of MF Prop.3.16.**b**!

Written assignment 5:
Prove MF Prop.3.16.**d**: Let $a, b, c \in R$. Then $a(bc) = c(ab)$, i.e., $a \odot (b \odot c) = c \odot (a \odot b)$.