Math 330 Section 2 - Fall 2018 - Homework 02

Published: Thursday, August 23, 2018   *Running total: 7 points*
Last submission: Friday, September 7, 2018

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
    Preface and ch.1

MF lecture notes:
    ch.1; ch.2.1 – 2.3; ch.3.1
    optional - needed for hwk 1: part of ch.3.2

New reading assignments:

OPTIONAL reading assignment (highly recommended)

   a. The following parts of the B/K lecture notes are an easy read which covers some of the properties
          of sets and functions. Read them if you feel wobbly on that subject!
   - ch.1.1 (Introduction to sets) (optional)
   - ch.1.1 (Introduction to sets) (optional) ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor
     and Ceiling Functions

Reading assignment 1 - due Monday, August 27:

   a. Read carefully MF ch.3.2 –3.3. Understand that this is the same material as B/G ch.1, except for
          the notation: $\mathbb{Z}$ in B/G vs. $(R, \oplus, \odot)$ in MF
   b. Read carefully B/G ch.2.1 – 2.2.

Reading assignment 2 - due: Wednesday, August 29:

   a. Read carefully the remainder of MF ch.3. Understand that MF 3.4 is the same material as B/G
          ch.2.1 – 2.2.
   b. Read carefully MF ch.2.4 (induction and recursion). Work through the examples! Finish MF ch.2.

Reading assignment 3 - due Friday, August 31:

   a. Read carefully the remainder of B/G ch.2. Understand that only with the induction axiom you
          have the true definition of the integers, and that integral domains need not satisfy it (and the
          well–ordering principle). Counterexample: $\mathbb{R}$.

> **General note on written assignments:** Unless expressly stated otherwise, to prove a proposition
> or theorem you are allowed to make use of everything in the book up to but NOT including the
> specific item you are asked to prove.

In the written assignments below R denotes an ordered integral domain $(R, \oplus, \odot, P)$.

Written assignment 1:
Use anything up-to and including MF prop. 3.31 to prove MF prop.3.32: The multiplicative unit 1 of R belongs to P.

Hint: This is an **indirect proof!** Part of it: Show that you cannot have $\ominus 1 \in P$. **Why** will this help you?

Written assignment 2:
Use anything up-to and including MF prop. 3.32 to prove MF prop.3.33: If $a \in R$ then $a \oplus 1 > a$.

GOOD NEWS: When you do assignments from MF ch.3.4 and later chapters or B/G ch.2 and later chapters, you do not need to justify the rules of arithmetic given to you in MF ch.3.3 and B/G ch.1. No more worry about commutativity of "$\oplus$" and "$\odot$" and the need for parentheses to group more than two terms. You may even use the "general laws of associativity": Given any finite sum of element of an integral domain such as $(a_1 \oplus a_2) \oplus (b_1 \oplus b_2)$ you may regroup the parentheses and even drop them. The same is true for products.