Math 330 Section 2 - Fall 2018 - Homework 05

Published: Thursday, August 23, 2018 Running total: 26 points Last submission: Friday, September 14, 2018 **NO RESUBMISSIONS** (same submission date as for hwk 4!)

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete so far (includes those of homework 4):

B/G (Beck/Geoghegan) Textbook: Preface, ch.1 – ch.3, ch.5

MF lecture notes:

ch.1 - ch.3, ch.5

B/K lecture notes (optional but very useful for hwk 3):

ch.1.1 (Introduction to sets)

ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments: None: They came with homework 4.

The written assignments are graded only once, and partial credit is given. The entire set is worth 6 points.

Written assignment 1:

Injectivity and Surjectivity

- Let $f: \mathbb{R} \longrightarrow [0, \infty[; x \mapsto x^2]$.
- Let $g:[0,\infty[\longrightarrow [0,\infty[; x\mapsto x^2]]$.

In other words, g is same function as f as far as assigning function values is concerned, but its domain was downsized to $[0, \infty[$.

Answer the following with true or false.

- **a.** f is surjective **c.** g is surjective
- **b.** f is injective **d.** g is injective

If your answer is **false** then give a specific counterexample.

Written assignment 2:

Find $f: X \longrightarrow Y$ and $A \subseteq X$ such that $f(A^{\complement}) \neq f(A)^{\complement}$. Hint: use $f(x) = x^2$ and choose Y as a **one element only** set (which does not leave you a whole lot of choices for X). See MF example 5.18 on approx. p.110. Start this problem as follows: Let $X := \{\ldots, X\}$, $X := \{\ldots, X\}$.

Written assignment 3:

Let
$$f:]-10, 10[\longrightarrow \mathbb{R}; \quad x \mapsto x^2.$$

a. what is the range of *f*? **b.** Is *f* injective? **c.** Is *f* surjective?

d.
$$f(\{1\} \cup [4,6]) = ?$$
 e. $f([2,5]) \cap f([4,7]) = ?$ **f.** $f^{-1}([4,25]) \cap f^{-1}([16,49]) = ?$

Written assignment 4:

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You have learned in MF ch.5 that injective \circ injective = injective, surjective \circ surjective = surjective.
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The following illustrates that the reverse is not necessarily true.

Find functions $f: \{a\} \longrightarrow \{b_1, b_2\}$ and $g: \{b_1, b_2\} \longrightarrow \{a\}$ such that $h:= g \circ f: \{a\}$ is bijective but such that it is **not true** that both f, g are injective and it is also **not true** that both f, g are surjective. Do not use any other sets when doing this problem!

Hint: There are not a whole lot of possibilities. Draw possible candidates for f and g in arrow notation as on p.118. You should easily be able to figure out some examples. Again, think simple and look at MF example 5.18 on approx. p.110.