Math 330 Section 2 - Fall 2018 - Homework 06

Published: Thursday, September 6, 2018 *Running total:* 30 *points*
Last submission: Wednesday, September 26, 2018

Update September 18, 2018

Last submission date was moved from 9/21/2018 to 9/26/2018. One point each for #1a and #1b, so total for this assignment is 4 points!

Status - Reading Assignments:

Here is the status of the reading assignments you were asked to complete by this date.

B/G (Beck/Geoghegan) Textbook:
Preface, ch.1 – ch.3, ch.5

MF lecture notes:
ch.1 – ch.3, ch.5

B/K lecture notes (optional but **very useful for hwk 3**):
ch.1.1 (Introduction to sets)
ch.1.2 (Introduction to Functions) but skip ch.1.2.4: Floor and Ceiling Functions

New reading assignments:

Reading assignment 1 - due Monday, September 10:

a. Read carefully MF ch.6.1, 6.2, 6.4. You may skip the optional ch.6.3 which just restates the contents of ch.6.2 for ordered integral domains, but be aware of what is in there as references will be made to that chapter when needed.
b. Read carefully the corresponding parts of B/G ch.4 and review what is written there and in earlier chapters about divisibility, in particular the proofs by induction given in B/G ch.2.

Reading assignment 2 - due: Wednesday, September 12:

a. Read carefully MF ch.6.5 and 6.6, and review B/G ch.2.3 and ch.2.4.
b. Read carefully B/G ch.4.3 (The Binomial Theorem).

Reading assignment 3 - due Friday, September 13:

a. Read carefully the end of B/G ch.4. Note that strong induction is also handled in MF ch.6.1.
b. Read carefully B/G ch.6.1. You have encountered most of that material in MF ch.5.1.
c. Read carefully B/G ch.6.2 and MF ch.6.7

General note on written assignments: Unless expressly stated otherwise, to prove a proposition or theorem you are allowed to make use of everything in the book up to but NOT including the specific item you are asked to prove.

Written assignment 1 – One point each for #1a and #1b:

Let X, Y be two nonempty sets and let $f : X \longrightarrow Y$. For $a, b \in X$ we write $a \sim b$ iff $f(a) = f(b)$.

- **a:** Prove that $\sim$ is an equivalence relation on X.
- **b:** Write $[x]_f$ for the equivalence class of $x \in X$ with respect to "$\sim$". Express $[x]_f$ in terms of the function f: $[x]_f = \{x' \in X : f(x') .....??.......\}$. (I do not want to see "$[x]_f = \{x' \in X : x' \sim x\}$".)

Written assignment 2:
Prove B/G Prop. 4.7(i) using induction: Let $k \in \mathbb{N}$. Then $5^{2k} - 1$ is divisible by 24.

You may use everything up to but not including B/G Prop.4.7.

Written assignment 3: Prove MF Prop. 6.3.1 by induction on c: Let $(x_j)_{j \in \mathbb{N}}$ be a sequence in $\mathbb{Z}$ and let $a, b, c \in \mathbb{Z}$ such that $a \leq b < c$. Then

$$\sum_{j=a}^{c} x_j = \sum_{j=a}^{b} x_j + \sum_{j=b+1}^{c} x_j.$$

Hints: Think carefully about the base case: If $a = 5$, how would you choose b and c? If $a = 28$, how would you choose b and c? For general a, how would you choose b and c?